Math 496/889 **Final** **Name:**______________________

Tuesday, December 16, 2003 **SSN:**______________________

Problem	1	2	3	4	5	6	Total
Possible	10	20	15	20	10	25	100
Points							

1. (10 points) Two call options are identical except they are written on two different stocks (stock 1 and stock 2) with different volatilities respectively σ_1 and σ_2, with $\sigma_1 < \sigma_2$. Which will be worth more? Explain why.

   The call option on stock 2 will be worth more, since Vega is always positive. As in the notes: "An increase in the volatility will lead to a corresponding increase in the call option value."

2. (20 points) Use the put-call parity relationship to derive the relationship between

   (a) The Delta of European call and the Delta of European put.

   (b) The Theta of European call and a European put.

   Show your complete work.

   (a) Differentiating the put-call parity with respect to S, we obtain the relationship between the Delta of a European call and the Delta of a European put:

$$\frac{\partial C}{\partial S} - \frac{\partial P}{\partial S} = 1$$

   or more compactly

$$\Delta_C - \Delta_P = 1.$$

   (b) Differentiating the put-call parity with respect to t, we obtain the relationship between the Theta of a European call and the Theta of a European put:

$$\frac{\partial C}{\partial t} - \frac{\partial P}{\partial t} = -rKe^{-r(T-t)}$$

   or more compactly

$$\Theta_C - \Theta_P = -rKe^{-r(T-t)}.$$

3. (15 points) A stock has a constant volatility of 18% and the risk-free interest rate (compounded continuously) is 6%. What is the value of an option to buy the stock for \$25 in two years time, given the current stock price is \$20?

Black-Scholes formula gives \$1.220977957.

$$\begin{aligned} d_1 &= -0.2779069159 \\ d_2 &= -0.5324653572 \\ Phi(d_1) &= 0.3905419074 \\ Phi(d_2) &= 0.292018647 \end{aligned}$$

4. (20 points) Suppose that a stock has an expected return of 16% per annum and a volatility of 30% per annum. When the stock price at the end of a certain day is \$50, calculate the following:

   (a) The expected stock price at the end of the next day.

   (b) The standard deviation of the stock price at the end of the next day.

$dS = rSdt + \sigma SdW = 0.16 \times 50 \times 1/365 + 0.30 \times 50 \times W_(1/365)$ *Therefore* $S_(1/365)$ *will be a random variable with mean* $50 + 0.0219$ *and standard deviation* $0.30 \times 50 \times sqrt 1/365 = 0.78513$

5. (10 points) A stock price is currently \$20. Tomorrow, important news is expected that will either immediately increase the price by \$5 or decrease the price by \$5. Discuss the merits of using the Black-Scholes formula to value options on the stock.

   The main problem is that this stock will not change continuously according to the model SDE for Geometric Brownian motion, and so the conclusions of the Black-Scholes model cannot be applied to this stock or its derivatives. It might make more sense in this contrived situation to use a binomial pricing model, but the situation is really just a thought-experiment to exercise your thinking about the underlying models.

6. (25 points) A company's cash position, measured in millions of dollars, follows a general Brownian motion with a drift rate of 0.1 per month, and a variance rate of 0.16 per month. The initial cash position is 2.0 That is, the cash position at time t follows the SDE

$$\begin{aligned} dX &= 0.1\,dt + 0.16\,dW \\ X(0) &= 2.0 \end{aligned}$$

   (a) What are the probability distributions of the cash position after 1 month, 6 months, and 1 year?

   (b) What are the probabilities of a negative cash position at the end of 6 months and one year?

   (a) The cash position at 1 month, 6 months, and 1 year are respectively

$$\begin{aligned} X(1) &\sim N(2.1, 0.16) \\ X(6) &\sim N(2.6, 0.96) \\ X(12) &\sim N(3.2, 1.92) \end{aligned}$$

   (b)

$$\begin{aligned} \Pr[X(1) < 0] &= \Pr[(Z - 2.1)/0.16 < -2.1/0.16] = 1.1E - 38 \\ \Pr[X(6) < 0] &= \Pr[(Z - 2.6)/0.96 < -2.6/0.96] = 0.00338 \\ \Pr[X(12) < 0] &= \Pr[(Z - 3.2)/1.92 < -3.2/1.92] = 0.04779 \end{aligned}$$