Project 1: Analyzing the Distribution of Stock Price Changes

Steve Dunbar
Math 489/889
Fall 2005

November 11, 2005

There are at least four schools of thought on the statistical distribution of stock price differences, or more generally, stochastic models for sequences of stock prices. In terms of followers, by far the most popular approach is that of the so-called "technical analyst", phrased in terms of short term trends, support and resistance levels, technical rebounds, and so on. Technical analysts believe that they can accurately predict the future price of a stock by looking at its historical prices and other trading variables. Technical analysis assumes that market psychology influences trading in a way that enables predicting when a stock will rise or fall. For that reason, many technical analysts are also market timers, who believe that technical analysis can be applied just as easily to the market as a whole as to an individual stock.

Rejecting this technical viewpoint, two other schools agree that sequences of prices are equivalent to a random walk. That is, price changes are independent of the previous price history, but these schools disagree in their choice of the appropriate probability distributions for the price changes. Some authors believe price changes have a normal distribution while the other group finds a distribution with "fatter tail probabilities", and perhaps even an infinite variance. Finally, a fourth group, overlapping with the previous two, admits the random walk as a first-order approximation but notes sizable second-order effects.

This project is to show a compatibility between the middle two groups. It has been noted that those that find price changes to be normally distributed to typically measure the changes over a fixed number of transactions, while those that find the larger tail probabilities typically measure price changes over a fixed time period that includes a random number of transactions. Use as the measure of fatness, (and there could be dispute over this) the *coefficient of excess*

$$\gamma_2 = [m_4/(m_2)^2] - 3$$

where m_k is the kth moment of the random variable about its mean.

The "tails" referred to in the previous paragraph is the part of the probability distribution form a to ∞, where a is an unspecified parameter. We say that one probability distribution (corresponding to a random variable X) has a "fatter tail" than another probability distribution (say corresponding to a random variable Y) if

$$\Pr[X \geq a] \geq \Pr[Y \geq a].$$

First we will investigate the meaning of "fatter tail probabilities" and its relationship to the coefficient of excess.

1. Consider

   (a) a standard normal random variable Z with pdf $f_Z(x) = e^{-x^2/2}/\sqrt{2\pi}$,

   (b) a "double exponential random variable" X with pdf $f_X(x) = (1/2)e^{-|x|}$ and

   (c) a standard uniform random variable U with pdf $f_U(x) = 1$ for $x = [-1/2, 1/2]$ and 0 elsewhere.

   (d) a Cauchy random variable Y with pdf $f_Y(x) = 1/(\pi(1 + x^2))$.

   On the same set of axes with a horizontal axis $-5 \leq x \leq 5$, graph the pdf of each of these random variables.

2. For each of the random variables Z, X, U and Y calculate the probability that the r.v. is greater than 1.

3. On the basis of the answers from the previous two questions, rank the random variables Z, X, U and Y in terms of the fatness of the tails.

4. Find the coefficient of excess $\gamma_2(Z)$ for a normal r.v. with pdf $f_Z(x) = e^{-x^2/2}/\sqrt{2\pi}$.

5. Find the coefficient of excess $\gamma_2(X)$ for a double exponential r.v. with pdf $f_X(x) = (1/2)e^{-|x|}$.

6. Find the coefficient of excess $\gamma_2(U)$ for a uniform r.v. with pdf $f_U(x) = 1$ for $x \in [-1/2, 1/2]$.

7. Find the coefficient of excess $\gamma_2(Y)$ for a Cauchy r.v. with pdf $f_Y(x) = 1/(\pi(1 + x^2))$.

8. Make a conjecture about the relation between "fatness of tails" and the coefficient of excess.