Stochastic Processes and
Advanced Mathematical Finance

Speculation and Hedging

Rating
Student: contains scenes of mild algebra or calculus that may require guidance.
Section Starter Question

Discuss examples of speculation in your experience. (Example: think of “scalping tickets”.) A hedge is a transaction or investment that is taken out specifically to reduce or cancel out risk. Discuss examples of hedges in your experience.

Key Concepts

1. Options have two primary uses, speculation and hedging.

2. Options can be a cheap way of exposing a portfolio to a large amount of risk. Sometimes a large amount of risk is desirable. This is the use of options and derivatives for speculation.

3. Options allow the investor to insure against adverse security value movements while still benefiting from favorable movements. This is use of options for hedging. This insurance comes at the cost of buying the option.

Vocabulary

1. Risk is random financial variation that has a known (or assumed) probability distribution. Uncertainty is chance variability that is due to unknown and unmeasured factors.

2. Speculation is to assume a financial risk in anticipation of a gain, especially to buy or sell to profit from market fluctuations.
3. **Hedging** is to protect oneself financially against loss by a counter-balancing transaction, especially to buy or sell assets as a protection against loss because of price fluctuation.

---

**Mathematical Ideas**

**Definitions**

Options have two primary uses, **speculation** and **hedging**. **Speculation** is to assume a financial risk in anticipation of a gain, especially to buy or sell to profit from market fluctuations. The market fluctuations are random financial variations with a known (or assumed) probability distribution.

**Risk and Uncertainty**

**Risk**, first articulated by the economist F. Knight in 1921, is a variability that you can put a price on. That is, **risk** is random financial variation that has a known (or assumed) probability distribution. In poker, say that you’ll win a poker hand unless your opponent draws to an inside straight, a particular kind of card draw from the deck. It is not necessary to know what this poker play means. However what is important to know is that this particular kind of draw has a probability of exactly 1/11. A poker player can calculate the 1/11 with simple rules of probability theory. Your bet is risk, you gain or lose of your bet with a known probability. It may be unpleasant to lose the bet, but at least you can account for it in advance with a probability.

**Uncertainty** is chance variability due to unknown and unmeasured factors. You might have some awareness (or not) of the variability out there. You may have no idea of how many such factors exist, or when any one may strike, or how big the effects will be. Uncertainty is the “unknown unknowns”.

---
Risk sparks a free-market economy with the impulse to make a gain. Uncertainty halts an economy with fear.

**Example: Speculation on a stock with calls**

An investor who believes that a particular stock, say XYZ, is going to rise may purchase some shares in the company. If she is correct, she makes money; if she is wrong she loses money. The investor is *speculating*. Suppose the price of the stock goes from $2.50 to $2.70, then the investor makes $0.20 on each $2.50 investment, or a gain of 8%. If the price falls to $2.30, then the investor loses $0.20 on each $2.50 share, for a loss of 8%. These are both standard calculations.

Alternatively, suppose the investor thinks that the share price is going to rise within the next couple of months, and that the investor buys a call option with exercise price of $2.50 and expiry date in three months.

Now assume that it costs $0.10 to purchase a European call option on stock XYZ with expiration date in three months and strike price $2.50. That means in three months time, the investor could, if the investor chooses to, purchase a share of XYZ at price $2.50 per share *no matter what the current price of XYZ stock is!* Note that the price of $0.10 for this option may not be an proper price for the option, but we use $0.10 simply because it is easy to calculate with. However, 3-month option prices are often about 5% of the stock price, so $0.10 is reasonable. In three months time if the XYZ stock price is $2.70, then the holder of the option may purchase the stock for $2.50. This action is called exercising the option. It yields an immediate profit of $0.20. That is, the option holder can buy the share for $2.50 and immediately sell it in the market for $2.70. On the other hand if in three months time, the XYZ share price is only $2.30, then it would not be sensible to exercise the option. The holder lets the option expire. Now observe carefully: By purchasing an option for $0.10, the holder can derive a net profit of $0.10 ($0.20 revenue less $0.10 cost) or a loss of $0.10 (no revenue less $0.10 cost.) The profit or loss is magnified to 100% with the same probability of change.

Investors usually buy options in quantities of hundreds, thousands, even tens of thousands so the absolute dollar amounts can be large. Compared with stocks, options offer a great deal of leverage, that is, large relative changes in value for the same investment. Options expose a portfolio to a large amount of risk cheaply. Sometimes a large degree of risk is desirable. This is the use of options and derivatives for speculation.
Example: Speculation on a stock with calls

Consider the profit and loss of an investor who buys 100 call options on XYZ stock with a strike price of $140. Suppose the current stock price is $138, the expiration date of the option is two months, and the option price is $5. Since the options are European, the investor can exercise only on the expiration date. If the stock price on this date is less than $140, the investor will choose not to exercise the option since buying a stock at $140 that has a market value less than $140 is not sensible. In these circumstances the investor loses the whole of the initial investment of $500. If the stock price is above $140 on the expiration date, the holder will exercise the options. Suppose for example, the stock price is $155. By exercising the options, the investor is able to buy 100 shares for $140 per share. By selling the shares immediately, the investor makes a gain of $15 per share, or $1500 ignoring transaction costs. Taking the initial cost of the option into account, the net profit to the investor is $10 per option, or $1000 on an initial investment of $500. Note that this calculation ignores any time value of money.

Example: Speculation on a stock with puts

Consider an investor who buys 100 European put options on XYZ with a strike price of $90. Suppose the current stock price is $86, the expiration date of the option is in 3 months and the option price is $7. Since the options are European, the holder will exercise only if the stock price is below $90 at the expiration date. Suppose the stock price is $65 on this date. The investor can buy 100 shares for $65 per share, and under the terms of the put option, sell the same stock for $90 to realize a gain of $25 per share, or $2500. Again, this simple example ignores transaction costs. Taking the initial cost of the option into account, the investor’s net profit is $18 per option, or $1800. This is a profit of 257% even though the stock has only changed price $25 from an initial of $90, or 28%. Of course, if the final price is above $90, the put option expires worthless, and the investor loses $7 per option, or $700.

Example: Hedging with calls on foreign exchange rates

Suppose that a U.S. company knows that it is due to pay 1 million pounds to a British supplier in 90 days. The company has significant foreign exchange
risk. The cost in U.S. dollars of making the payment depends on the exchange rate in 90 days. The company instead can buy a call option contract to acquire 1 million pounds at a certain exchange rate, say 1.7 in 90 days. If the actual exchange rate in 90 days proves to be above 1.7, the company exercises the option and buys the British pounds it requires for $1,700,000. If the actual exchange rate proves to be below 1.7, the company buys the pounds in the market in the usual way. This option strategy allows the company to insure itself against adverse exchange rate increases but still benefit from favorable decreases. Of course this insurance comes at the relatively small cost of buying the option on the foreign exchange rate.

Example: Hedging with a portfolio with puts and calls

Since the value of a call option rises when an asset price rises, what happens to the value of a portfolio containing both shares of stock of XYZ and a negative position in call options on XYZ stock? If the stock price is rising, the call option value will also rise, the negative position in calls will become greater, and the net portfolio should remain approximately constant if the positions are in the right ratio. If the stock price is falling then the call option value price is also falling. The negative position in calls will become smaller. If held in the proper amounts, the total value of the portfolio should remain constant! The risk (or more precisely, the variation) in the portfolio is reduced! The reduction of risk by taking advantage of such correlations is called hedging. Used carefully, options are an indispensable tool of risk management.

Consider a stock currently selling at $100 and having a standard deviation in its price fluctuations of $10, for a proportion variation of 10%. We can use the Black-Scholes formula derived later to show that a call option with a strike price of $100 and a time to expiration of one year would sell for $11.84. A 1 percent rise in the stock from $100 to $101 would drive the option price to $12.73. Consider the total effects in Table 1.

Suppose a trader has an original portfolio comprised of 8944 shares of stock selling at $100 per share. (The unusual number of 8944 shares comes from the Black-Scholes formula as a hedge ratio.) Assume also that a trader short sells call options on 10,000 shares at the current price of $11.84. That is, the short seller borrows the options from another trader and therefore must later return the options at the option price at the return time. The obligation to return the borrowed options creates a negative position in the
option value. The transaction is called short selling because the trader sells a good he or she does not actually own and must later pay it back. In Table 1 this debt or short position in the option is indicated by a minus sign. The entire portfolio of shares and options has a net value of $776,000.

Now consider the effect of a 1 percent change in the price of the stock. If the stock increases 1 percent, the shares will be worth $903,344. The option price will increase from $11.84 to $12.73. But since the portfolio also involves a short position in 10,000 options, this creates a loss of $8,900. This is the additional value of what the borrowed options are now worth, so the borrower must additionally this amount back! Taking these two effects into account, the value of the portfolio will be $776,044. This is nearly the same as the original value. The slight discrepancy of $44 is rounding error due to the fact that the number of stock shares calculated from the hedge ratio is rounded to an integer number of shares for simplicity in the example, and the change in option value is rounded to the nearest penny, also for simplicity. In actual practice, financial institutions take great care to avoid round-off differences.

On the other hand of the stock price falls by 1 percent, there will be a loss in the stock of $8944. The price on this option will fall from $11.84 to $10.95 and this means that the entire drop in the price of the 10,000 options will be $8900. Taking both of these effects into account, the portfolio will then be worth $776,956. The overall value of the portfolio will not change (to within $44 due to round-off effects) regardless of what happens to the stock price. If the stock price increases, there is an offsetting loss on the option; if the stock price falls, there is an offsetting gain on the option.

This example is not intended to illustrate a prudent investment strategy. If an investor desired to maintain a constant amount of money, putting the sum of money invested in shares into the bank or in Treasury bills instead would safeguard the sum and even pay a modest amount of interest. If the investor wished to maximize the investment, then investing in stocks solely and enduring a probable 10% loss in value would still leave a larger total investment.

This example is a first example of short selling. It is also an illustration of how holding an asset and short selling a related asset in carefully calibrated ratios can hold a total investment constant. The technique of holding and short-selling to keep a portfolio constant will later be an important idea in deriving the Black-Scholes formula.
Original Portfolio  \( S = 100, C = \$11.84 \)

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>8,944 shares of stock</td>
<td>$894,400</td>
</tr>
<tr>
<td>Short position on 10,000 options</td>
<td>-$118,400</td>
</tr>
<tr>
<td>Net value</td>
<td>$776,000</td>
</tr>
</tbody>
</table>

| Stock Price rises 1%   | \( S = 101, C = \$12.73 \)
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>8,944 shares of stock</td>
<td>$903,344</td>
</tr>
<tr>
<td>Short position on 10,000 options</td>
<td>-$127,300</td>
</tr>
<tr>
<td>Net value</td>
<td>$776,044</td>
</tr>
</tbody>
</table>

| Stock price falls 1%   | \( S = 99, C = \$10.95 \)
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>8,944 shares of stock</td>
<td>$885,456</td>
</tr>
<tr>
<td>Short position on options</td>
<td>-$109,500</td>
</tr>
<tr>
<td>Net value</td>
<td>$775,956</td>
</tr>
</tbody>
</table>

Table 1: Hedging to keep a portfolio constant.

Sources


Problems to Work for Understanding

1. You would like to speculate on a rise in the price of a certain stock. The current stock price is $29 and a 3-month call with strike of $30 costs $2.90. You have $5,800 to invest. Identify two alternate strategies, one involving investment in the stock, and the other involving investment...
in the option. What are the potential gains or losses from each due to a rise to $31 in three months? What are the potential gains or losses from each due to a fall to $27 in three months?

2. A company knows it is to receive a certain amount of foreign currency in 4 months. What kind of option contract is appropriate for hedging? What is the risk? Be specific.

3. The current price of a stock is $94 and 3-month call options with a strike price of $95 currently sell for $4.70. An investor who feels that the price of the stock will increase is trying to decide between buying 100 shares and buying 2,000 call options. Both strategies involve an investment of $9,400. Write and solve an inequality to find how high the stock price must rise for the option strategy to be the more profitable. What advice would you give?

---

**Reading Suggestion:**

**References**


Outside Readings and Links:

- Speculation and Hedging A short youtube video on speculation and hedging, from “The Trillion Dollar Bet”.
- More Speculation and Hedging A short youtube video on speculation and hedging.

I check all the information on each page for correctness and typographical errors. Nevertheless, some errors may occur and I would be grateful if you would alert me to such errors. I make every reasonable effort to present current and accurate information for public use, however I do not guarantee the accuracy or timeliness of information on this website. Your use of the information from this website is strictly voluntary and at your risk.

I have checked the links to external sites for usefulness. Links to external websites are provided as a convenience. I do not endorse, control, monitor, or guarantee the information contained in any external website. I don’t guarantee that the links are active at all times. Use the links here with the same caution as you would all information on the Internet. This website reflects the thoughts, interests and opinions of its author. They do not explicitly represent official positions or policies of my employer.

Information on this website is subject to change without notice.

Steve Dunbar’s Home Page, http://www.math.unl.edu/~sdunbar1
Email to Steve Dunbar, sdunbar1 at unl dot edu
Last modified: Processed from \LaTeX{} source on July 11, 2016