Math 489/889
Stochastic Processes and
Advanced Mathematical Finance
Homework

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Due Wed, November 10, 2010

1. If you buy a lottery ticket in 50 independent lotteries, and in each lottery your chance of winning a prize is $1/100$, write down and evaluate the probability of winning and also approximate the probability using the Central Limit Theorem.

   (a) exactly one prize,
   (b) at least one prize,
   (c) at least two prizes.

   Explain with a reason whether or not you expect the approximation to be a good approximation.

2. A bank has $1,000,000 available to make for car loans. The loans are in random amounts uniformly distributed from $5,000 to $20,000. Make a model for the total amount that the bank loans out. How many loans can the bank make with 99% confidence that it will have enough money available?

3. Let $W(t)$ be standard Brownian motion.

   (a) Find the probability that $0 < W(1) < 1$.
   (b) Find the probability that $0 < W(1) < 1$ and $1 < W(2) - W(1) < 3$. 

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(c) Find the probability that \(0 < W(1) < 1\) and \(1 < W(2) - W(1) < 3\) and \(0 < W(3) - W(2) < 1/2\).

4. Let \(W(t)\) be standard Brownian motion.

(a) Evaluate the probability that \(W(5) \leq 3\) given that \(W(1) = 1\).

(b) Find the number \(c\) such that \(\Pr[W(9) > c | W(1) = 1] = 0.10\).

5. Use your Homework 5 record of a 100-flip coin flip sequence. Scoring \(Y_i = +1\) for each Head and \(Y_i = -1\) for each Tail on each flip, keep track of the accumulated sum \(T_n = \sum_{i=1}^{n} Y_i\) for \(n = 1, \ldots, 100\) representing the net fortune at any time. Plot the resulting \(T_n\) versus \(n\) on the interval \([0, 100]\). Finally, using \(N = 10\) plot the rescaled approximation \(W_{10}(t) = (1/\sqrt{10})S(10t)\) on the interval \([0, 10]\) on the same set of axes.

6. Let \(Z\) be a single normally distributed random variable, with mean 0 and variance 1, i.e. \(Z \sim N(0, 1)\). Then consider the continuous time stochastic process \(X(t) = \sqrt{t}Z\).

(a) Using a normal random variable generator (from Excel, Maple, Mathematica, Octave, MATLAB, R etc., all have one and probably the TI-89 or equivalent has one too), find sample values of \(X(1), X(2), X(4)\) and \(X(9)\).

(b) Explain why the distribution of \(X(t)\) is normal with mean 0 with variance \(t\).

(c) Is \(X(t)\) a Brownian motion? Explain why or why not.