Math 489/889

Stochastic Processes and

Advanced Mathematical Finance

Homework 1

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Due Friday, September 3, 2010

Problem 1

part a

Find and write the definition of a ``future", also called a futures contract. Graph the intrinsic value of a futures contract at its contract date, or expiration date, as was done for the call option.

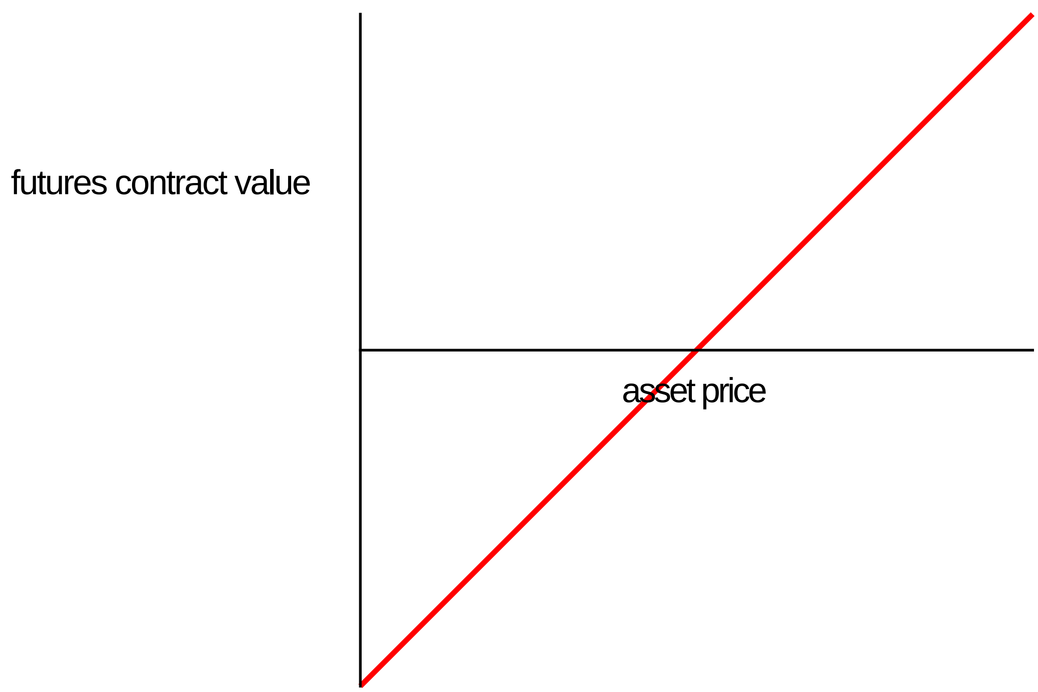
From investorwords.com "A standardized, transferable, exchange-traded contract that requires delivery of a commodity, bond, currency, or stock index, at a specified price, on a specified future date. Unlike options, futures convey an obligation to buy. The risk to the holder is unlimited, and because the payoff pattern is symmetrical, the risk to the seller is unlimited as well. Dollars lost and gained by each party on a futures contract are equal and opposite. In other words, futures trading is a zero-sum game. Futures contracts are forward contracts, meaning they represent a pledge to make a certain transaction at a future date. The exchange of assets occurs on the date specified in the contract. "

From Wikipedia.org "Futures contract, in finance, refers to a standardized contract to buy or sell a specified commodity of standardized quality at a certain date in the future, at a market determined price (the futures price). The contracts are traded on a futures exchange. Futures contracts are not "direct" securities like stocks, bonds, rights or warrants as outlined by the Uniform Securities Act. They are still securities, however, though they are a type of derivative contract. ... In many cases, the underlying asset to a futures contract may not be traditional "commodities" at all – that is, for financial futures, the underlying asset or item can be currencies, securities or financial instruments and intangible assets or referenced items such as stock indexes and interest rates.

The future date is called the delivery date or final settlement date. ...

A futures contract gives the holder the obligation to make or take delivery under the terms of the contract, whereas an option grants the buyer the right, but not the obligation, to establish a position previously held by the seller of the option. In other words, the owner of an options contract may exercise the contract, but both parties of a "futures contract" must fulfill the contract on the settlement date. The seller delivers the underlying asset to the buyer, or, if it is a cash-settled futures contract, then cash is transferred from the futures trader who sustained a loss to the one who made a profit."

$plot(S - 15, S = 0 ..30, scaling = constrained, labels = [\text{"asset price"}, \text{"futures contract value"}], tickmarks = [0, 0], thickness = 2)$

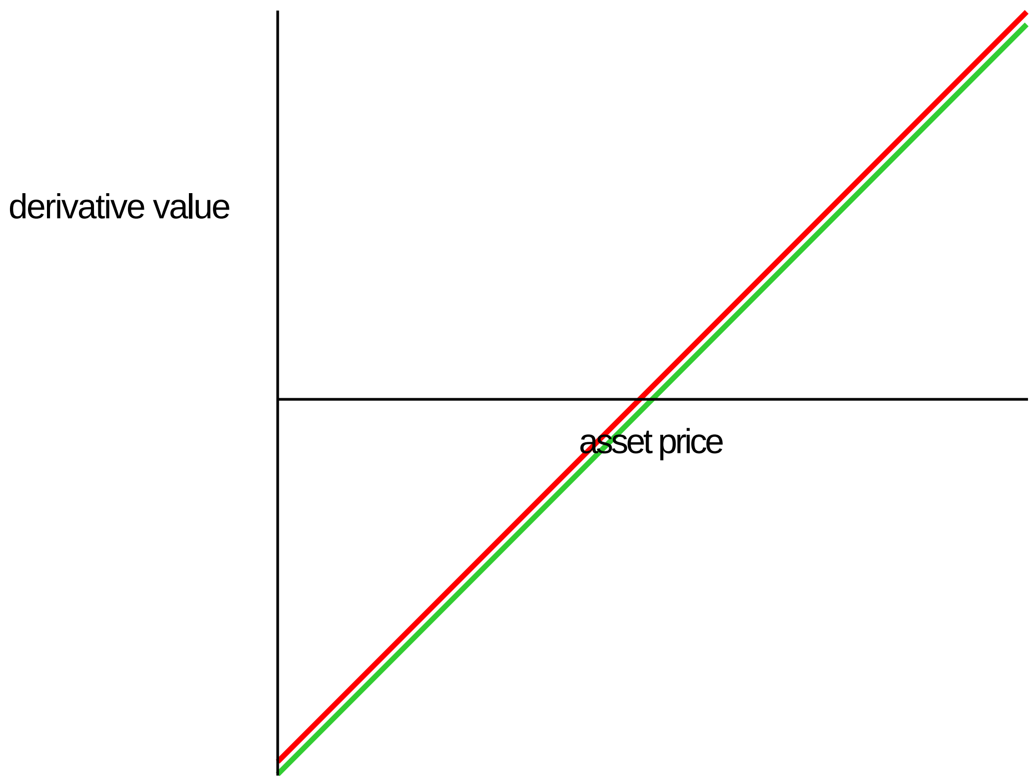


Part b

Show that holding a call option and writing a put option on the same asset, with the same strike price K is the same as having a futures contract on the asset with strike price K. Drawing a graph of the value of the combination and the value of the futures contract together with an explanation will demonstrate the equivalence.

Note that in the following plot, I shift the futures contract value up by 0.5 just to make it visible alongside the plot of the value of the option combination. This is for graphing and visibility purposes only, it has nothing to do with the valuation of the contract.

$plot([S-15+0.5, \max(0, S-15)-\max(15-S, 0)], S=0..30, scaling=constrained, labels=["asset price", "derivative value"], tickmarks=[0, 0], thickness=[2, 2])$



Problem 2

Puts and calls are not the only option contracts available, just the most fundamental and the simplest. Puts and calls are designed to eliminate risk of up or down price movements in the underlying asset. Some other option contracts designed to eliminate other risks are created as combinations of puts and calls.;

Long Strangle

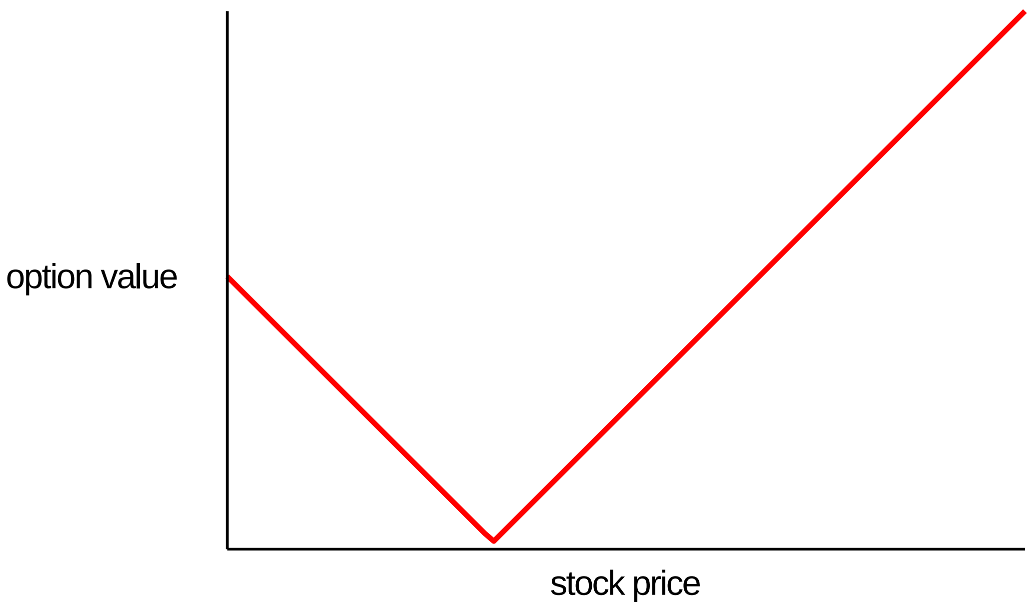
Draw the graph of the value of the option contract composed of holding a put option with strike price K_1 and holding a call option with strike price K_2 where $K_1 < K_2$ (Assume both the put and the call have the same expiration date.) The investor profits only if the underlier moves dramatically in either direction. This is known as a **long strangle**.

$plot(\ \max(\ 10 - S, 0, S - 20), S = 0..30, scaling = constrained, labels = [\text{"stock price"}, \text{"option value"}], tickmarks = [0, 0], thickness = 2\ )$

Long Strangle (Bull Straddle)

Draw the graph of the value of an option contract composed of holding a put option with strike price *K* and holding a call option with the same strike price *K*. (Assume both the put and the call have the same expiration date.) This is called an **long straddle** and also called a **bull straddle**.

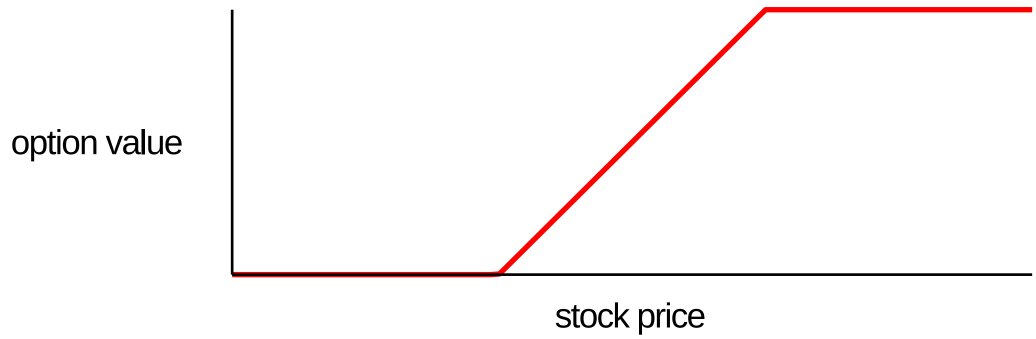
$plot(\ \max(\ 10 - S, S - 10), S = 0..30, scaling = constrained, labels = [\text{"stock price"}, \text{"option value"}], tickmarks = [0, 0], thickness = 2\ )$



Bull call spread

Draw the graph of the value of an option contract composed of holding one call option with strike price K_1 and the simultaneous writing of a call option with strike price K_2 with $K_1 < K_2$. (Assume both the options have the same expiration date.) This is known as a **bull call spread**.

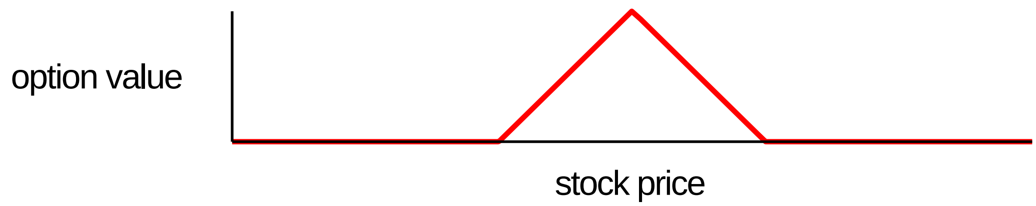
$plot(\max(S-10,0)+(-\max(0,S-20)), S=0..30, scaling=constrained, labels=["stock price", "option value"], tickmarks=[0,0], thickness=2)$



Butterfly spread

Draw the graph of the value of an option contract created by simultaneously holding one call option with strike price K_1, holding another call option with strike price K_2 where $K_1 < K_2$, and writing two call options at strike price $(K_1 + K_2)/2$. This is known as a **butterfly spread**.

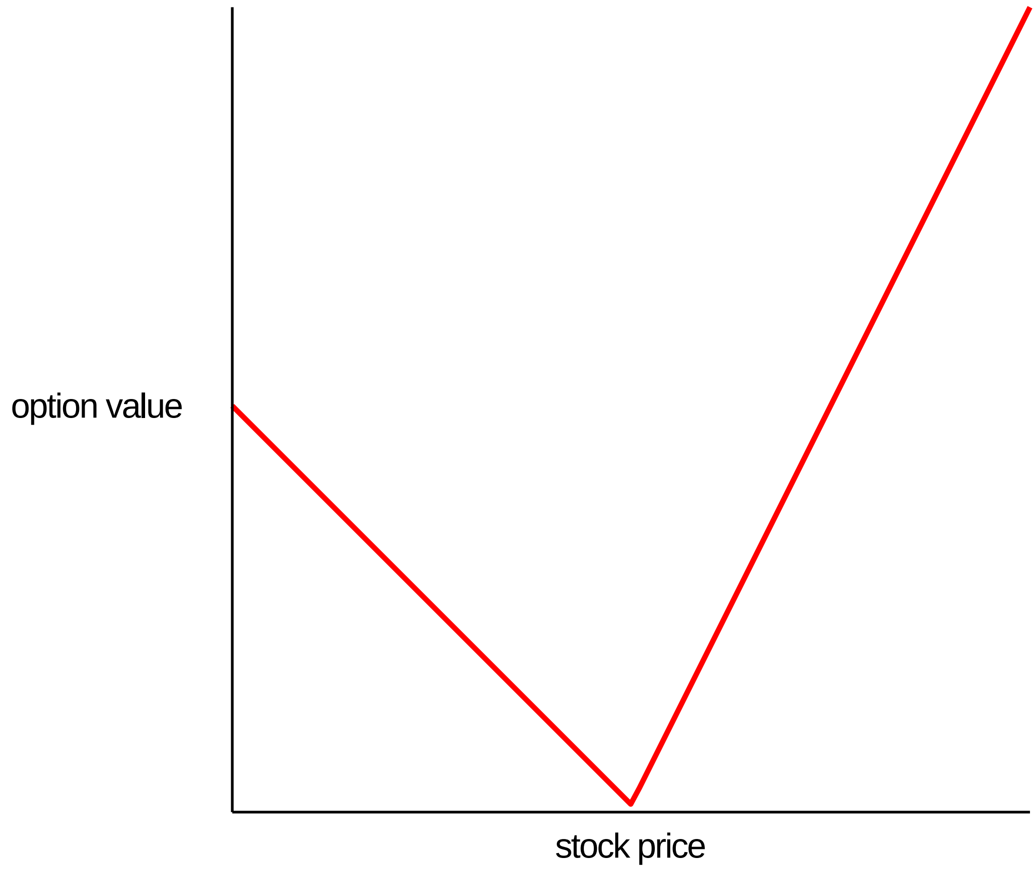
$$plot(\ \max(\ S - 10, 0)\ + \max(0, S - 20)\ - 2\cdot\max(0, S - 15), S = 0..30, scaling = constrained,$$
$$labels = [\text{"stock price"}, \text{"option value"}], tickmarks = [0, 0], thickness = 2\ )$$



Triple option

Draw the graph of the value of an option contract created by holding one put option with strike price *K* and holding two call options on the same underlying security, strike price, and maturity date. This is known as a **triple option** or **strap.**

$$plot(\ \max(10 - S, 0)\ + 2\cdot\max(0, S - 10), S = 0..20, scaling = constrained, labels = [\text{"stock price"}, \text{"option value"}], tickmarks = [0, 0], thickness = 2\ )$$



Problem 3

You would like to speculate on a rise in the price of a certain stock. The current stock price is $29 and a 3-month call with strike of $30 costs $2.90. You have $5,800 to invest. Identify two alternative strategies, one involving investment in the stock, and the other involving investment in the option. What are the potential gains and losses from each?

Let X be amount of your $5,800 invested in stock, so that 5,800-X would be the amount invested in calls. Then X/29 is the number of shares of stack you can buy and (5800 - X)/2.90 is the number of calls you can buy. Let S be the stock price at the end of the 3-months. Then the value of what you hold is:

$$\frac{X}{29} \cdot S + \frac{(5800 - X)}{2.90} \cdot \max(0, S - 30)$$

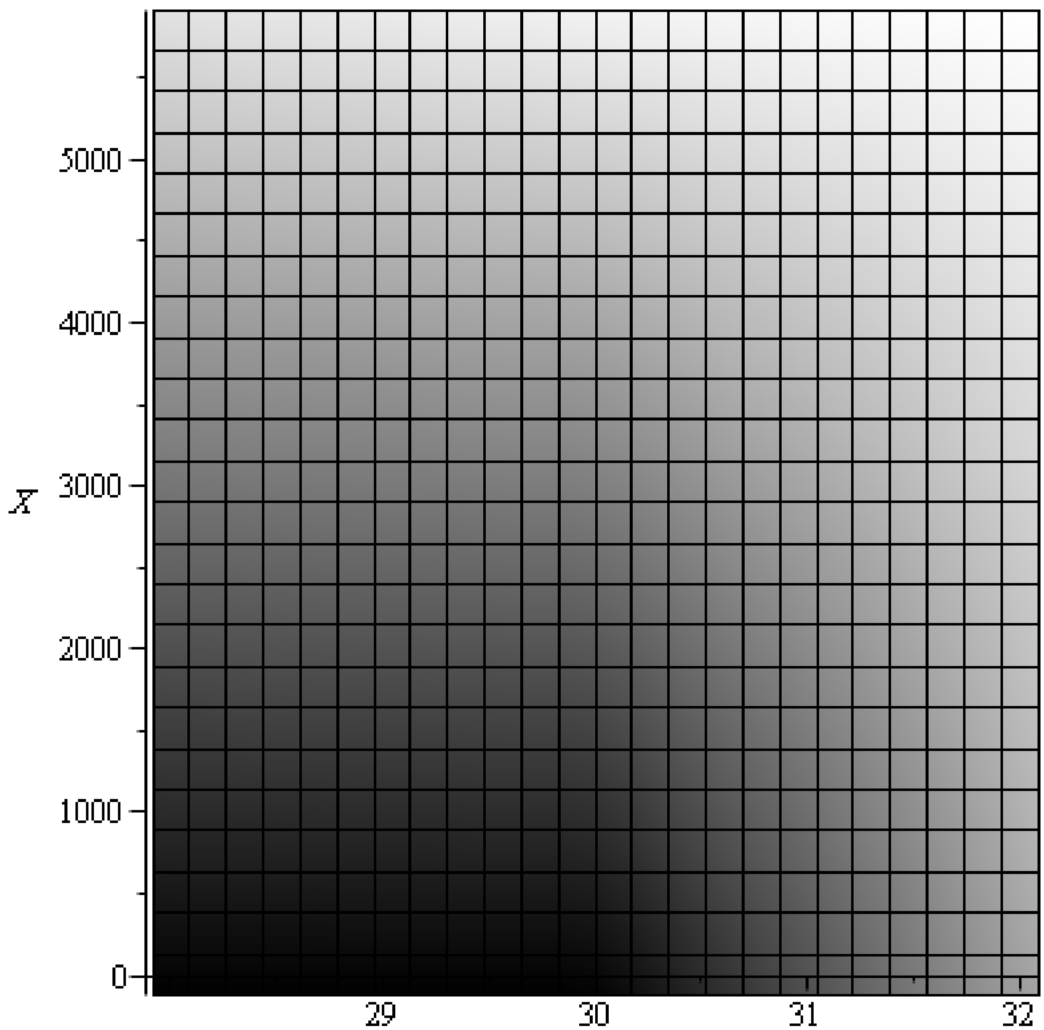
$$\frac{1}{29} X S + 0.3448275862 \, (5800 - X) \max(0, S - 30) \quad \textbf{(3.1)}$$

Your profit or loss is the value, less what you paid:

$$\left(\frac{X}{29}\cdot S - X\right) + \frac{(5800 - X)}{2.90}\cdot \max(0, S - 30) - (5800 - X)$$

I'm going to look at this in a density plot, which is like a contour plot over the independent variables X and S, using darkness to indicate value instead of plotting contour lines. Darker means lower values, lighter means higher values.

$$\left(\frac{X}{29}\cdot S - X\right) + \frac{(5800 - X)}{2.90}\cdot \max(0, S - 30) - (5800 - X) \rightarrow$$



For a given stock value (plotted for reasonable values between 28 and 32) the lighter colors always appear at the value X = 5800. So this indicates that for greatest profit (or least loss), invest all in stocks, none in options.

Problem 4

A company knows it is to receive a certain amount of foreign currency in 4 months. What type of option contract is appropriate for hedging? Please be very specific.

To be specific, suppose a U.S. company is to receive payment of 1 million Euros in 4 months. The company will then convert to US dollars. The compnay could buy a **foreign exchage put option** allowing them to sell Euros at a certain conversion rate to US dollars, say a strike of 1.35 dollars for every Euro. If the exchange rate falls below the strike, then by exercising the option the company can sell the Euros and receive $1,350,000. If the exchange rate goes above this strike then the company will not exercise the option, sell the Euros at the existing rate and receive more than $1,350,000. Of course this insurance comes at the cost of purchasing the option.

Problem 5

The current price of a stock is $94 and 3-months call options with a strike price of $95 currently sell for $4.70. An investor who feels that the price of the stock will increase is trying to decide between buying 100 shares and buying 2,000 call options. Both strategies involve an investment of $9,400. What advice would you give? How high does the stock price have to rise for the option strategy to be the more profitable?

Let X be amount of your $9,400 invested in stock, so that 9,400-X would be the amount invested in calls. Then X/94 is the number of shares of stack you can buy and (9400 - X)/4.70 is the number of calls you can buy. Let S be the stock price at the end of the 3-months. Then the value of what you hold is:

$$\frac{X}{94} \cdot S + \frac{(9400 - X)}{4.70} \cdot \max(0, S - 95)$$

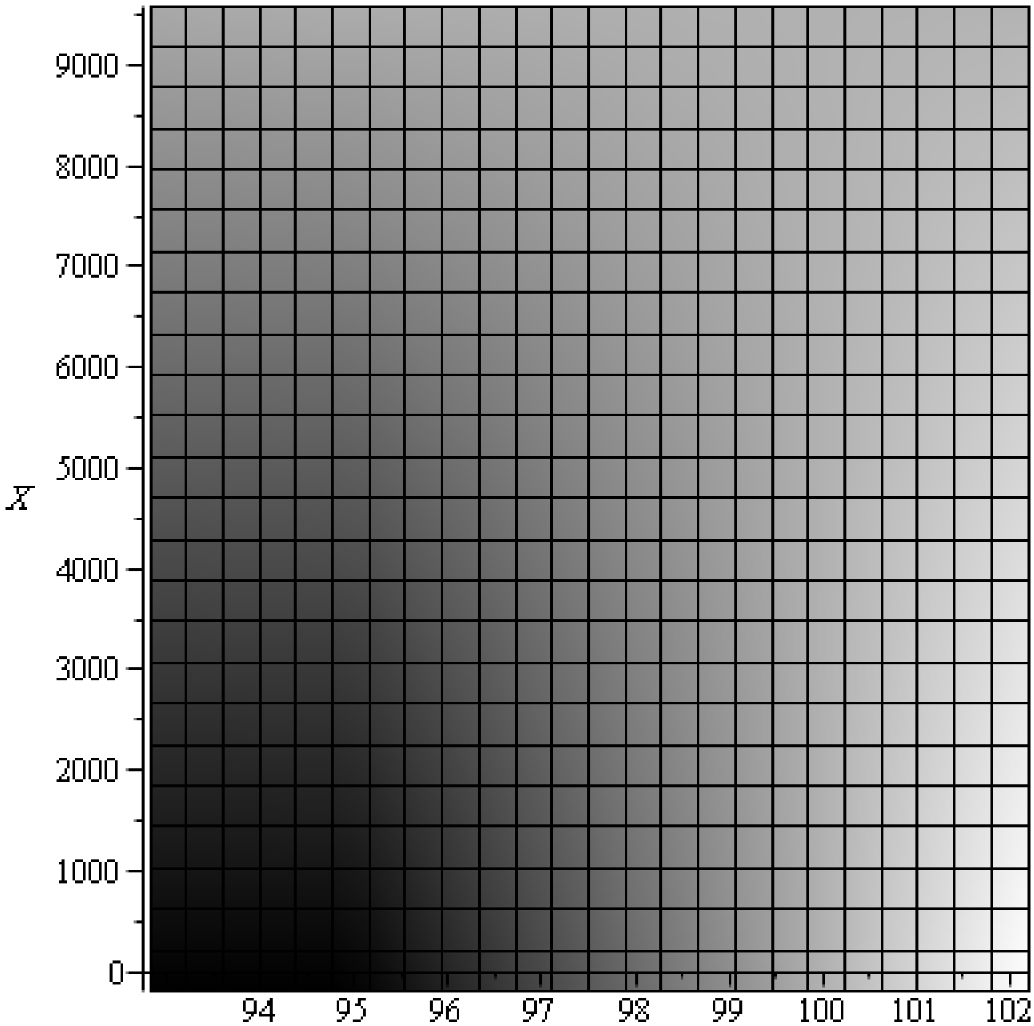
$$\frac{1}{94} X S + 0.2127659574\,(9400 - X)\,\max(0, S - 95) \qquad \textbf{(5.1)}$$

Your profit or loss is the value, less what you paid:

$$\left(\frac{X}{94} \cdot S - X\right) + \frac{(9400 - X)}{4.70} \cdot \max(0, S - 95) - (9400 - X)$$

I'm going to look at this in a density plot, which is like a contour plot over the independent variables X and S, using darkness to indicate value instead of plotting contour lines. Darker means lower values, lighter means higher values.

$$\left(\frac{X}{94} \cdot S - X\right) + \frac{(9400 - X)}{4.70} \cdot \max(0, S - 95) - (9400 - X) \rightarrow$$



$$\left(\frac{X}{94}\cdot S - X\right) + \frac{(9400 - X)}{4.70}\cdot \max(0, S - 95) - (9400 - X) \rightarrow$$

Maple is unable to render 3D graphics.

Your operating system, graphics, or video driver may require updating.

See "gldriver" in the help system for more information.

GLException

Error making context current

From the plot, it appears that the stock proce has to rise above about S = 100 for the options ot be more profitable