1. (a) For $T_0 = 10$ and $a = 20$, draw a graph of the expected duration of a coin-flipping game until victory or ruin as a function of the probability $q$.

(b) For $a = 20$ and $q = 0.55$ draw a graph of the expected duration of a coin-flipping game until victory or ruin as a function of $T_0$.

(c) For $a = 20$ and $q = 0.45$ draw a graph of the expected duration of a coin-flipping game until victory or ruin as a function of $T_0$.

2. Find a particular solution $W_{sk}^p$ to the non-homogeneous equation

$$W_{sk}^p = \delta_{sk} + \frac{1}{2}W_{s-1,k}^p + \frac{1}{2}W_{s+1,k}^p.$$ 

using the trial function

$$W_{sk}^p = \begin{cases} 0 & \text{if } s \leq k \\ E(s - k) & \text{if } s > k. \end{cases}$$
3. Show that

\[ W_s = \sum_{k=1}^{s-1} kW_{sk} \]

\[ = 2 \left[ \frac{s}{S} \sum_{k=1}^{s-1} k(S - k) - \sum_{k=1}^{s-1} k(s - k) \right] \]

\[ = 2 \left[ \frac{s}{S} \left[ \frac{S(S-1)(S+1)}{6} \right] - \frac{s(s-1)(s+1)}{6} \right] \]

\[ = \frac{s}{3} [S^2 - s^2] \]

You will need formulas for \( \sum_{k=1}^{N} k \) and \( \sum_{k=1}^{N} k^2 \) or alternatively for \( \sum_{k=1}^{N} k(M - k) \). These are easily found or derived.

4. (a) For the long run average cost

\[ C = K + \frac{(1/3)r S^3 x(1 - x^2)}{S^2 x(1 - x)}. \]

find \( \partial C/\partial x \).

(b) For the long run average cost

\[ C = K + \frac{(1/3)r S^3 x(1 - x^2)}{S^2 x(1 - x)}. \]

find \( \partial C/\partial S \).

(c) Find the optimum values of \( x \) and \( S \).