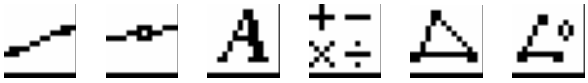
EXPLORATIONS ™

Activity 4

Objective

- To measure the interior and exterior angles of a triangle and find their relationships

Cabri® Jr. Tools

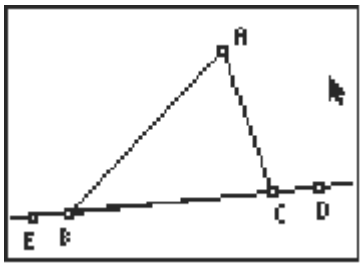


Angles of a Triangle

Introduction

Interior angles of a triangle are the angles formed at each vertex of the triangle that lie inside the triangle. There are three interior angles of ΔABC in the figure, $\angle ABC$, $\angle ABC$, and $\angle BAC$.

Exterior angles of a triangle are the angles formed by extending the sides of the triangle. The exterior angles $\angle ACD$ and $\angle ABE$ are shown in the figure when side $\overline{BC}$ is extended. If sides $\overline{AC}$ and $\overline{AB}$ are extended, four more exterior angles are formed.



Adjacent angles are angles with a common vertex and a common side that have no interior region in common. In the figure, $\angle ACB$ and $\angle ACD$ are adjacent angles.

Supplementary angles are two angles with measurements that sum to 180 degrees.

Remote interior angles of a triangle are the interior angles not adjacent to the exterior angle. Sometimes these angles are referred to as the non-adjacent interior angles. In the figure, $\angle ABC$ and $\angle BAC$ are remote interior angles of exterior angle $\angle ACD$.

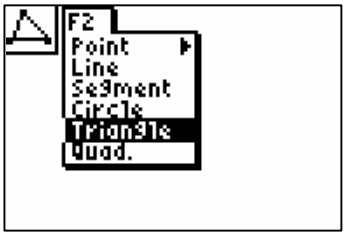
Construction & Exploration

Part I: Interior angles of a triangle

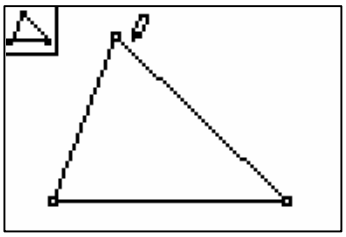
Draw $\angle ABC$ to fill most of the screen, as in the figure above.

1. Press WINDOW to view the **Drawing Tools Menu** (**F2 Menu**) and then highlight **Triangle**. Press ENTER.

   Note: *The tool icon at the top left of the screen indicates that the* ***Triangle*** *tool is active.*

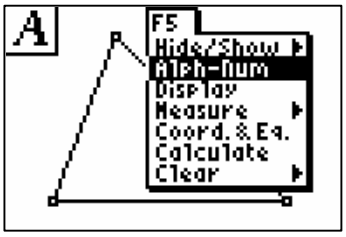


2. Move the cursor to the lower left corner of the screen, and press ENTER to anchor the first vertex of the triangle.

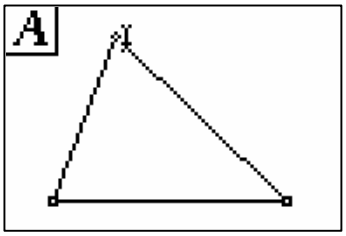


3. Move the cursor to the desired location for the second vertex, and press ENTER to anchor it. Repeat this for the third vertex.

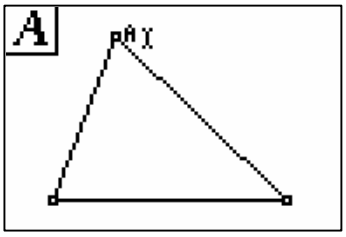
4. Press GRAPH to view the **Display Tools Menu** (**F5 Menu**) and then highlight **Alpha-Num**. Press ENTER.



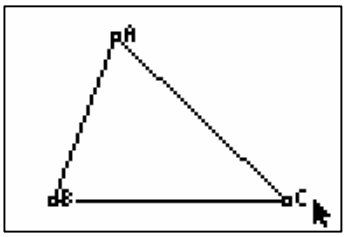
5. Move the cursor to the highest vertex of the triangle. The point blinks when the cursor is close enough to the point to select it.



6. Press ENTER to create a label for this point. Press MATH to label that point *A* (note that alpha lock is on), and then press ENTER to complete the label.

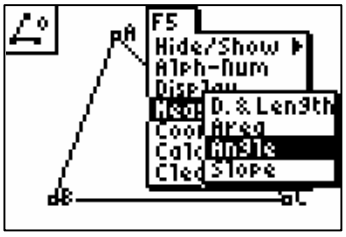


7. Repeat step **6** to label the other vertices *B* and *C* (B is above APPS, C is located above PRGM). When complete, press CLEAR to exit the **Alpha-Numeric Tool**.



Measure the three interior angles of $\angle ABC$.

8. Press GRAPH to view the **Display Tools Menu** (**F5 Menu**) and then highlight **Measure**. Press ▶ to view the **Measure Menu**. Highlight **Angle**, and press ENTER.



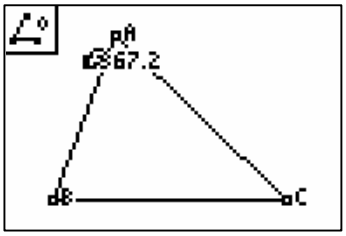
9. Angles are measured by selecting three points. Select the vertex point of the angle *second*.

a. To measure $\angle A$, first move the cursor to point *B*, and press ENTER.

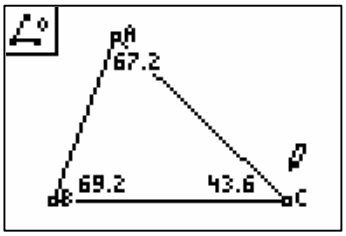
b. Move the cursor to the vertex of the angle, point *A*, and press ENTER.

c. Move the cursor to the third point on the angle, point *C*, and press ENTER.

d. Finally, move the measurement to a desired location, and press ENTER to anchor it on the screen.



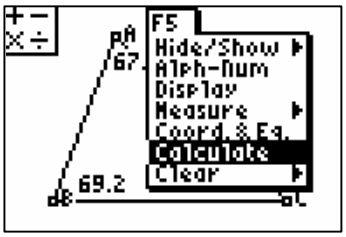
Repeat this step for $\angle B$ and $\angle C$. When complete, press CLEAR to exit the **Angle Measure** tool.



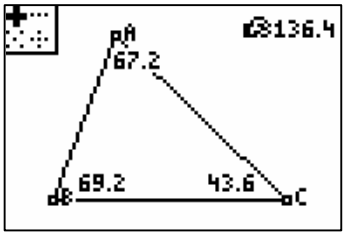
Note: *The graphing handheld application measures each angle to two decimal places, but displays each measurement to only one decimal place. Therefore, the displayed angle measurement may appear to be inaccurate.*

Calculate the sum of the interior angles.

10. Press GRAPH to view the **Display Tools Menu** (**F5 Menu**), and then highlight **Calculate**. Press ENTER.



11. A calculation is performed on a pair of numbers. Start by adding the measures of $\angle A$ and $\angle B$.



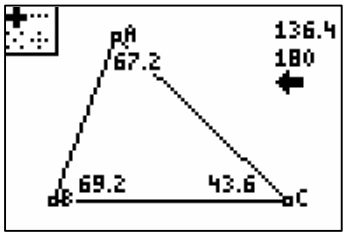
a. Move the cursor to the measure of $\angle A$, and press ENTER.

b. Press + to indicate addition.

c. Move the cursor to the measure of $\angle B$, and press ENTER.

d. Use the cursor keys to move the calculation to a blank area of the screen, and press ENTER a final time to anchor the calculation.

12. Now add this sum to the measure of $\angle C$.



a. Highlight the sum of the first two angles, and press ENTER.

b. Press + to indicate addition.

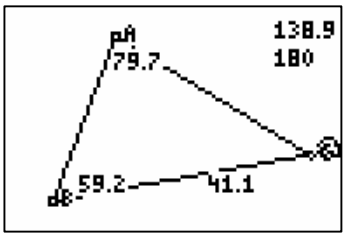
c. Move the cursor to the measure of $\angle C$, and press ENTER.

d. Use the cursor keys to move the calculation to a blank area of the screen, and press ENTER a final time to anchor the calculation.

When complete, press CLEAR to exit the **Calculate** tool.

Drag a vertex of an angle.

13. Move the cursor to highlight the desired vertex, and press [ALPHA] to grab it. Use the cursor keys to drag the vertex around the screen, and observe what happens to the angle measures and their sum.



Note: *Make sure to grab the point itself, not the labels A, B, or C.*

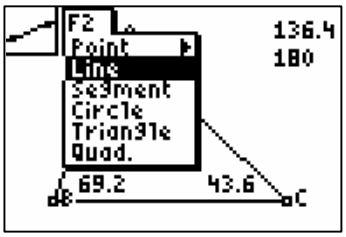
Note: *The pointer changes to a hollow arrow when it is near an available object.*

When complete, press [CLEAR] to exit.

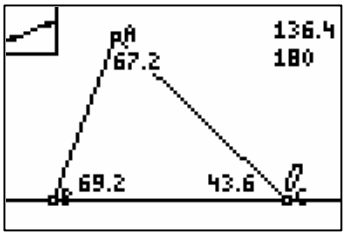
Part II: Exterior angles of a triangle

Construct a line on one side of the triangle.

1. Press [WINDOW] to view the **Drawing Tools Menu** (**F2 Menu**), and then highlight **Line**. Press [ENTER].



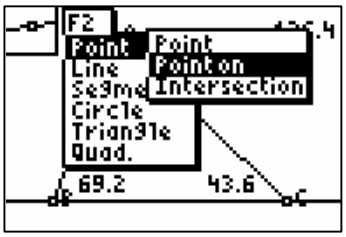
2. Move the cursor to vertex *B* of the triangle, and press [ENTER] to anchor one point of the line.



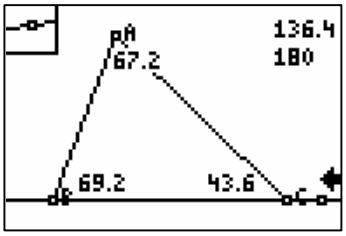
3. Move the cursor to vertex *C* of the triangle, and press [ENTER] to anchor the second point of the line.

Construct and label a point on the line to the right of the triangle.

4. Press [WINDOW] to view the **Drawing Tools Menu** (**F2 Menu**), and then highlight **Point**. Press [▶] to view the **Point Menu**. Highlight **Point on**, and press [ENTER].

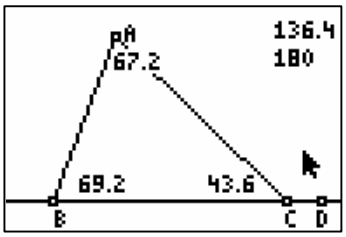


5. Move the cursor to the right of the triangle on the line, and press [ENTER] to place a point on the line.



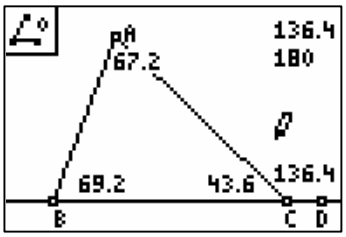
6. Use the **Alpha-Numeric** tool ([GRAPH], **F5 Menu**) to label the newly created point as *D* (D is above [x^{-1}]). Remember to press [ENTER] once to create the label, and again to complete the label.

   Note: *If necessary, press [ALPHA] to grab and move labels and other objects to a more convenient area.*

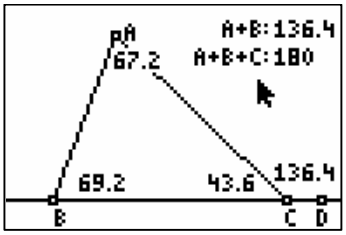


Measure $\angle ACD$, and make calculations.

7. Use the **Angle Measure** tool (GRAPH, **F5 Measure Menu**) to measure exterior angle $\angle ACD$.



8. Use the **Calculate** tool (GRAPH, **F5 Menu**) to calculate the sum of exterior angle $\angle ACD$ and the adjacent interior angle $\angle ACB$.



9. Use the **Calculate** tool (GRAPH, **F5 Menu**) to calculate the sum of the remote (non-adjacent) interior angles $\angle A$ and $\angle ABC$.

10. If desired, the sums can be labeled using the **Alpha-Numeric** tool (GRAPH **F5 Menu**). Move the cursor to the left of the desired sum, and press ENTER once to create the label.

Note: *Remember that* [A-LOCK] *is active, so if you want to type the symbol "+", press* ALPHA *to turn off the* [A-LOCK]*, press* +*, and then press* ALPHA *to turn* [A-LOCK] *back on.*

11. Press ALPHA to grab and drag a vertex of the triangle, and observe what happens to the sums.

Data Collection and Analysis

Name _____

Date _____

Questions and Conjectures

1. What is the sum of the measures of the interior angles of a triangle?

_____

2. Does it matter whether the triangle is acute or obtuse? Explain your reasoning.

_____

_____

3. What is the sum of the exterior angle and its adjacent interior angle?

_____

4. What is the sum of the two remote (non-adjacent) interior angles?

_____

5. Can the exterior angle be made larger than either remote (non-adjacent) interior angle? Smaller? Explain.

_____

_____

_____

Extension

6. Construct and measure an exterior angle at each of the other two vertices of the triangle. What is the sum of the three exterior angles of the triangle? Does it matter whether the triangle is acute or obtuse? Explain.

_____

_____

_____

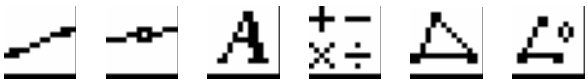
Teacher Notes

Activity 4

Objective

- To measure the interior and exterior angles of a triangle and find their relationships

Cabri® Jr. Tools



Angles of a Triangle

Answers to Questions and Conjectures

1. What is the sum of the measures of the interior angles of a triangle?

Dragging a vertex of the triangle should suggest that the sum of the angles of a triangle is equal to 180°. Students should note that the calculated sum of *A* and *B* does change (step **11**), but the sum of all three interior angles does not change (step **12**).

2. Does it matter whether the triangle is acute or obtuse? Explain your reasoning.

No; the sum is 180° regardless of the type of triangle. This is true even though a vertex was dragged to create a different type of triangle.

3. What is the sum of the exterior angle and its adjacent interior angle?

The sum of the interior angle and its adjacent exterior angle of a triangle is always equal to 180° since these two adjacent angles form a straight angle.

4. What is the sum of the two remote (non-adjacent) interior angles?

The sum of the remote (non-adjacent) interior angles will equal the exterior angle. The reason for this relationship is a combination of the angle sum theorem and the supplementary angles theorem just explored.

5. Can the exterior angle be made larger than either remote (non-adjacent) interior angle? Smaller? Explain.

The exterior angle *must* be larger than either remote (non-adjacent) interior angle since it will equal the sum of the two nonadjacent (remote) interior angles. However, the exterior angle *cannot* be smaller than either of the remote (non-adjacent) interior angles for the same reason.

Answer to Extension

6. Construct and measure an exterior angle at each of the other two vertices of the triangle. What is the sum of the three exterior angles of the triangle? Does it matter whether the triangle is acute or obtuse? Explain.

The sum of three exterior angles of a triangle (one at each vertex) will equal 360°. This is true regardless of the type of triangle. This relationship can be proved by adding the measures of six angles, three exterior (call them *x*, *y*, and *z*) and three interior (call them *a*, *b*, and *c*). Each pair of exterior and adjacent interior will equal 180° (since these two adjacent angles form a straight angle); therefore, three pairs will equal 540°. Subtracting three interior angles (which equal 180°) leaves three exterior angles with a sum of 360°.

$(x + a) + (y + b) + (z + c) = 3(180°)$

$(x + y + z) + (a + b + c) = 540°$

$(x + y + z) = 360°$