Section 5.1:

(16) The columns of the given matrix $A$ are easily seen to be orthonormal. Thus, the matrix is orthogonal. By Theorem 5.5,

$$A^{-1} = A^T = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$  

(19) Let $A$ be the given matrix. Then

$$A^T A = \begin{bmatrix} \cos \theta \sin \theta & \cos^2 \theta & \sin \theta \\ -\cos \theta & \sin \theta & 0 \\ -\sin^2 \theta & -\cos \theta \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} \cos \theta \sin \theta & -\cos \theta & -\sin^2 \theta \\ \cos^2 \theta & \sin \theta & -\cos \theta \sin \theta \\ \sin \theta & 0 & \cos \theta \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

To see this, you will need to use the trig identity $\sin^2 \theta + \cos^2 \theta = 1$. So, for example,

$$\cos^2 \theta \sin^2 \theta + \cos^4 \theta + \sin^2 \theta = \cos^2 \theta (\sin^2 \theta + \cos^2 \theta) + \sin^2 \theta$$
$$= \cos^2 \theta + \sin^2 \theta = 1$$

$$-\cos \theta \sin^3 \theta - \cos^3 \theta \sin \theta + \cos \theta \sin \theta = \cos \theta \sin \theta(-\sin^2 \theta - \cos^2 \theta + 1) = 0$$

We see that $A^{-1} = A^T$. So, by Theorem 5.5 $A$ is orthogonal and

$$A^{-1} = A^T = \begin{bmatrix} \cos \theta \sin \theta & \cos^2 \theta & \sin \theta \\ -\cos \theta & \sin \theta & 0 \\ -\sin^2 \theta & -\cos \theta \sin \theta & \cos \theta \end{bmatrix}.$$
(20) The columns of the given matrix $A$ are easily seen to be orthonormal. Thus, the matrix is orthogonal. By Theorem 5.5,

$$A^{-1} = A^T = \begin{bmatrix} 1/2 & 1/2 & -1/2 & 1/2 \\ -1/2 & 1/2 & 1/2 & 1/2 \\ 1/2 & 1/2 & 1/2 & -1/2 \\ 1/2 & -1/2 & 1/2 & 1/2 \end{bmatrix}.$$

(26) Suppose $Q$ is an orthogonal matrix. Let $P$ be a matrix obtained from $Q$ by interchanging rows of $Q$.

Since $Q$ is orthogonal, the row vectors of $Q$ form an orthonormal set (see Theorem 5.7). Since changing the order of a list of vectors doesn’t change the length of the vectors nor the orthogonality of the set, the row vectors of $P$ form an orthonormal set. Thus, the column vectors of $P^T$ form an orthonormal set. We conclude, using the definition of orthogonal matrices, that $P^T$ is an orthogonal matrix. Applying Theorem 5.5 to $P^T$ yields that

$$(P^T)^{-1} = (P^T)^T = P.$$ 

Therefore,

$$P^{-1} = [(P^T)^{-1}]^{-1} = P^T.$$ 

Now applying Theorem 5.5 to $P$ shows that $P$ is an orthogonal matrix.

**Section 5.2:**

(1) Observe that

$$W = \left\{ \begin{bmatrix} x \\ 2x \end{bmatrix} \in \mathbb{R}^2 \right\} = \text{span} \left( \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right).$$

That is $W$ equals the column space of the $2 \times 1$ matrix

$$A = \begin{bmatrix} 1 \\ 2 \end{bmatrix}.$$ 

By Theorem 5.10, $W^\perp = \text{null}(A^T)$. We have

$$A^T = \begin{bmatrix} 1 & 2 \end{bmatrix}.$$
After we solve the system $A^T x = 0$, we see that

\[ W^\perp = \text{null}(A^T) = \left\{ x = \begin{bmatrix} -2t \\ t \end{bmatrix} : t \in \mathbb{R} \right\} = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : x + 2y = 0 \right\} \]

Thus, a basis for $W^\perp$ is

\[ B^\perp = \left\{ \begin{bmatrix} -2 \\ 1 \end{bmatrix} \right\}. \]

(2) Observe that

\[ W = \left\{ \begin{bmatrix} x \\ (-3/4)x \end{bmatrix} \in \mathbb{R}^2 \right\} = \text{span} \left( \begin{bmatrix} 1 \\ -3/4 \end{bmatrix} \right). \]

That is $W$ equals the column space of the $2 \times 1$ matrix

\[ A = \begin{bmatrix} 1 \\ -3/4 \end{bmatrix}. \]

By Theorem 5.10, $W^\perp = \text{null}(A^T)$. We have

\[ A^T = \begin{bmatrix} 1 & -3/4 \end{bmatrix}. \]

After we solve the system $A^T x = 0$, we see that

\[ W^\perp = \text{null}(A^T) = \left\{ x = \begin{bmatrix} (3/4)t \\ t \end{bmatrix} : t \in \mathbb{R} \right\} = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : x - (3/4)y = 0 \right\} \]

Thus, a basis for $W^\perp$ is

\[ B^\perp = \left\{ \begin{bmatrix} 3/4 \\ 1 \end{bmatrix} \right\}. \]
(3) Observe that
\[
W = \left\{ \begin{bmatrix} x \\ y \\ x + y \end{bmatrix} \in \mathbb{R}^3 \right\} = \text{span} \left( \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right).
\]

That is \( W \) equals the column space of the \( 3 \times 2 \) matrix
\[
A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}.
\]

By Theorem 5.10, \( W^\perp = \text{null}(A^T) \). We have
\[
A^T = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}.
\]

After we solve the system \( A^T x = 0 \), we see that
\[
W^\perp = \text{null}(A^T) = \left\{ x = \begin{bmatrix} -t \\ -t \\ t \end{bmatrix} : t \in \mathbb{R} \right\}
\]
\[
= \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} : x = t, y = t, z = -t \right\}
\]

Thus, a basis for \( W^\perp \) is
\[
B^\perp = \left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \right\}.
\]

(6) Observe that
\[
W = \left\{ \begin{bmatrix} 2t \\ 2t \\ -t \end{bmatrix} \in \mathbb{R}^3 \right\} = \text{span} \left( \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix} \right).
\]
That is $W$ equals the column space of the $3 \times 1$ matrix

$$A = \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}.$$  

By Theorem 5.10, $W^\perp = \text{null}(A^T)$. We have

$$A^T = \begin{bmatrix} 2 & 2 & -1 \end{bmatrix}.$$  

After we solve the system $A^T \mathbf{x} = \mathbf{0}$, we see that

$$W^\perp = \text{null}(A^T) = \left\{ \mathbf{x} = \begin{bmatrix} -s + (t/2) \\ s \\ t \end{bmatrix} : s, t \in \mathbb{R} \right\} = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} : x = -s + (t/2), y = s, z = t \right\}.$$  

Thus, a basis for $W^\perp$ is

$$B^\perp = \left\{ \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1/2 \\ 0 \\ 1 \end{bmatrix} \right\}.$$  

(7) We begin by finding RREF($A$):

$$A \rightarrow \text{RREF}(A) = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$  

We see that a basis for $\text{row}(A)$ is

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix} \right\}.$$
By solving the system \( A\mathbf{x} = \mathbf{0} \), we find that \( x_1 = -x_3 = -t, x_2 = 2x_3 = 2t, x_3 = t \). So, a basis for \( \text{null}(A) \) is

\[
\left\{ \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} \right\}.
\]

Note that

\[
1(-1) + (0)(2) + (1)(1) = 0,
\]
\[
(0)(-1) + (1)(2) + (-2)(1) = 0.
\]

That is, the basis vectors for \( \text{row}(A) \) are orthogonal to the basis vectors for \( \text{null}(A) \). This is enough to show that every vector in \( \text{row}(A) \) is orthogonal to every vector in \( \text{null}(A) \).

(9) Using the RREF(A) from exercise 7, we see that a basis for \( \text{col}(A) \) is

\[
\left\{ \begin{bmatrix} 1 \\ 5 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 1 \\ -1 \end{bmatrix} \right\}.
\]

To find a basis for \( \text{null}(A^T) \), we need to row-reduce \( A^T \):

\[
A^T = \begin{bmatrix} 1 & 5 & 0 & -1 \\ -1 & 2 & 1 & -1 \\ 3 & 1 & -2 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 5 & 0 & -1 \\ 0 & 7 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}.
\]

By solving \( A^T \mathbf{x} = \mathbf{0} \), we find \( x_1 = 5/7t - 3/7s, x_2 = -t/7 + 2/7s, x_3 = t, x_4 = s \). So, a basis for \( \text{null}(A^T) \) is

\[
\left\{ \begin{bmatrix} 5 \\ -1 \\ 7 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 2 \\ 0 \\ 7 \end{bmatrix} \right\}.
\]
Note that

\[
(1)(5) + (5)(-1) + (0)(7) + (-1)(0) = 0 \\
(-1)(5) + (2)(-1) + (1)(7) + (-1)(0) = 0 \\
(1)(-3) + (5)(2) + (0)(0) + (-1)(7) = 0 \\
(-1)(-3) + (2)(2) + (1)(0) + (-1)(7) = 0
\]

That is, the basis vectors for \( \text{col}(A) \) are orthogonal to the basis vectors for \( \text{null}(A^T) \). This is enough to show that every vector in \( \text{col}(A) \) is orthogonal to every vector in \( \text{null}(A^T) \).

(11) We have that

\[
W = \text{span} \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}, \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix} = \text{col}(A)
\]

where

\[
A = \begin{bmatrix} 2 & 4 \\ 1 & 0 \\ -2 & 1 \end{bmatrix}
\]

So, by Theorem 5.10, \( W^\perp = \text{null}(A^T) \). We have

\[
A^T = \begin{bmatrix} 2 & 1 & -2 \\ 4 & 0 & 1 \end{bmatrix} \longrightarrow \begin{bmatrix} 2 & 1 & -2 \\ 0 & 2 & -5 \end{bmatrix}
\]

Thus, when we solve the system \( A^T \mathbf{x} = \mathbf{0} \), we have \( x_1 = -1/4t, x_2 = 5/2t, x_3 = t \). So, a basis for \( W^\perp = \text{null}(A^T) \) is

\[
\begin{pmatrix} 1 \\ -10 \\ -4 \end{pmatrix}
\]

(12) We have that

\[
W = \text{span} \begin{pmatrix} 1 \\ -1 \\ 3 \\ -2 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ -2 \\ 1 \end{pmatrix} = \text{col}(A)
\]

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where
\[
A = \begin{bmatrix}
1 & 0 \\
-1 & 1 \\
3 & -2 \\
-2 & 1
\end{bmatrix}.
\]

So, by Theorem 5.10, \(W^\perp = \text{null}(A^T)\). We have
\[
A^T = \begin{bmatrix}
1 & -1 & 3 & -2 \\
0 & 1 & -2 & 1
\end{bmatrix}.
\]

Thus, when we solve the system \(A^T x = 0\), we have \(x_1 = x_2 - 3x_3 + 2x_4 = -s + t, x_2 = 2x_3 - x_4 = 2s - t, x_3 = s, x_4 = t\). So, a basis for \(W^\perp = \text{null}(A^T)\) is
\[
\left\{ \begin{bmatrix}
-1 \\
2 \\
1 \\
0
\end{bmatrix}, \begin{bmatrix}
1 \\
-1 \\
0 \\
1
\end{bmatrix} \right\}.
\]

(16) By definition,
\[
\text{proj}_W(v) = \left( \frac{u_1 \cdot v}{u_1 \cdot u_1} \right) u_1 + \left( \frac{u_2 \cdot v}{u_2 \cdot u_2} \right) u_2.
\]

We calculate
\[
\begin{align*}
u_1 \cdot v &= 2 \\
u_2 \cdot v &= 2 \\
u_1 \cdot u_1 &= 3 \\
u_2 \cdot u_2 &= 2
\end{align*}
\]

Thus,
\[
\text{proj}_W(v) = \frac{2}{3} u_1 + \frac{2}{2} u_2 = \begin{bmatrix}
5/3 \\
-1/3 \\
2/3
\end{bmatrix}.
\]
(17) By definition, 

$$proj_W(v) = \left( \frac{u_1 \cdot v}{u_1 \cdot u_1} \right) u_1 + \left( \frac{u_2 \cdot v}{u_2 \cdot u_2} \right) u_2.$$ 

We calculate

\[
\begin{align*}
  u_1 \cdot v &= 1 \\
  u_2 \cdot v &= 13 \\
  u_1 \cdot u_1 &= 9 \\
  u_2 \cdot u_2 &= 18
\end{align*}
\]

Thus,

$$proj_W(v) = \frac{1}{9} u_1 + \frac{13}{18} u_2 = \begin{bmatrix} -1/2 \\ 1/2 \\ 3 \end{bmatrix}.$$ 

(21) Let $u_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ and $u_2 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$. Then $W = span(u_1, u_2)$. We want to write $v$ as

$$v = w + w^\perp,$$

where $w \in W$ and $w^\perp \in W^\perp$. Using the proof of Theorem 5.11, we see that

$$w = proj_W(v) = \left( \frac{u_1 \cdot v}{u_1 \cdot u_1} \right) u_1 + \left( \frac{u_2 \cdot v}{u_2 \cdot u_2} \right) u_2.$$ 

We calculate

\[
\begin{align*}
  u_1 \cdot v &= 3 \\
  u_2 \cdot v &= 9 \\
  u_1 \cdot u_1 &= 6 \\
  u_2 \cdot u_2 &= 3
\end{align*}
\]

Thus,

$$w = proj_W(v) = \frac{1}{2} u_1 + 3 u_2 = \begin{bmatrix} 7/2 \\ -2 \\ 7/2 \end{bmatrix}.$$
We now let
\[ w^\perp = v - w = \begin{bmatrix} 1/2 \\ 0 \\ -1/2 \end{bmatrix}. \]
So, the orthogonal decomposition of \( v \) with respect to \( W \) is
\[ v = w + w^\perp = \begin{bmatrix} 7/2 \\ -2 \\ 7/2 \end{bmatrix} + \begin{bmatrix} 1/2 \\ 0 \\ -1/2 \end{bmatrix}. \]

(25) No, it is not necessarily true that \( w' \) is in \( W^\perp \). For example, consider the subspace \( W \) of \( \mathbb{R}^3 \) from exercise 21. That is, let
\[ W = \text{span} \left( \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \right). \]
Take
\[ v = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \in \mathbb{R}^3. \]
We observe that since \( W \) is a subspace, the vector
\[ w := \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \]
is in \( W \). Moreover, we can write
\[ v = 0 + v. \]
Here the vector \( v \) is also playing the role of \( w' \). However,
\[ \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = 6 \neq 0. \]
This shows that \( w' \) is not in \( W^\perp \).
To do this exercise, it is helpful to use a part of exercise (27); namely, if $u \in W$, then $\text{proj}_W(u) = u$. Let’s see why this is true.

By Theorem 5.11 and its proof, we can write

$$u = w + w^\perp = \text{proj}_W(u) + w^\perp,$$

where $w = \text{proj}_W(u) \in W$ and $w^\perp \in W^\perp$ and these vectors are unique. But $0 \in W^\perp$ and $u \in W$ and

$$u = u + 0.$$

Since the vectors $w$ and $w^\perp$ are unique, we must have that $w = \text{proj}_W(u) = u$ and $w^\perp = 0$. This shows that $\text{proj}_W(u) = u$.

Now let $x$ be a vector in $\mathbb{R}^n$. Then, by definition, $\text{proj}_W(x)$ is a linear combination of vectors in $W$ and so $\text{proj}_W(x)$ is a vector in $W$ (since $W$ is a subspace and thus closed under scalar multiplication and vector addition!). But the projection of any vector in $W$ is just itself by the above observations. Therefore, we conclude that

$$\text{proj}_W(\text{proj}_W(x)) = \text{proj}_W(x).$$

Section 5.3:

(1) Let $v_1 = x_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. Then let

$$v_2 = x_2 - \frac{(v_1 \cdot x_2)}{(v_1 \cdot v_1)} v_1$$

$$= \begin{bmatrix} 1 \\ 2 \end{bmatrix} - \frac{3}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1/2 \\ 1/2 \end{bmatrix}$$

The set $\{v_1, v_2\}$ is an orthogonal basis for $\mathbb{R}^2$.

To obtain an orthonormal basis for $\mathbb{R}^2$ we normalize the vectors $v_1$ and $v_2$. We find

$$||v_1|| = \sqrt{v_1 \cdot v_1} = \sqrt{2}$$

$$||v_2|| = \sqrt{v_2 \cdot v_2} = 1/\sqrt{2}$$

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Let
\[ w_1 = \frac{1}{||v_1||} v_1 = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix} \]
and
\[ w_2 = \frac{1}{||v_2||} v_2 = \begin{bmatrix} -\sqrt{2}/2 \\ \sqrt{2}/2 \end{bmatrix}. \]

Then the set \{w_1, w_2\} is an orthonormal basis for \(\mathbb{R}^2\).

(4) Let \(v_1 = x_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\). Then let
\[ v_2 = x_2 - \left( \frac{v_1 \cdot x_2}{v_1 \cdot v_1} \right) v_1 \]
\[ = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} - \frac{2}{3} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \]
\[ = \begin{bmatrix} 1/3 \\ 1/3 \\ -2/3 \end{bmatrix} \]
and
\[ v_3 = x_3 - \left( \frac{v_1 \cdot x_3}{v_1 \cdot v_1} \right) v_1 - \left( \frac{v_2 \cdot x_3}{v_2 \cdot v_2} \right) v_2 \]
\[ = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} - \frac{1}{3} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \frac{1}{3} \begin{bmatrix} 1/3 \\ 2/3 \\ -2/3 \end{bmatrix} \]
\[ = \begin{bmatrix} 1/2 \\ -1/2 \\ 0 \end{bmatrix} \]

The set \(\{v_1, v_2, v_3\}\) is an orthogonal basis for \(\mathbb{R}^3\).
To obtain an orthonormal basis for $\mathbb{R}^3$ we normalize the vectors $v_1, v_2$ and $v_3$. We find

$$
||v_1|| = \sqrt{v_1 \cdot v_1} = \sqrt{3}
$$
$$
||v_2|| = \sqrt{v_2 \cdot v_2} = \sqrt{6}/3
$$
$$
||v_3|| = \sqrt{v_3 \cdot v_3} = \sqrt{2}/2
$$

Let

$$
w_1 = \frac{1}{||v_1||} v_1 = \begin{bmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix}
$$

and

$$
w_2 = \frac{1}{||v_2||} v_2 = \begin{bmatrix} 1/\sqrt{6} \\ 1/\sqrt{6} \\ -2/\sqrt{6} \end{bmatrix}
$$

and

$$
w_3 = \frac{1}{||v_3||} v_3 = \begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \\ 0 \end{bmatrix}
$$

Then the set $\{w_1, w_2, w_3\}$ is an orthonormal basis for $\mathbb{R}^3$.

(5) Let $v_1 = x_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$. Then let

$$
v_2 = x_2 - \left( \frac{v_1 \cdot x_2}{v_1 \cdot v_1} \right) v_1
$$

$$
= \begin{bmatrix} 3 \\ 4 \\ 2 \end{bmatrix} - \frac{7}{2} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}
$$

$$
= \begin{bmatrix} -1/2 \\ 1/2 \\ 2 \end{bmatrix}
$$
Then the set \( \{v_1, v_2\} \) is an orthogonal basis for \( W \).

(6) Let \( v_1 = x_1 = \begin{bmatrix} 2 \\ -1 \\ 1 \\ 2 \end{bmatrix} \). Then let

\[
v_2 = x_2 - \left( \frac{v_1 \cdot x_2}{v_1 \cdot v_1} \right) v_1
\]

\[
= \begin{bmatrix} 3 \\ -1 \\ 0 \\ 4 \end{bmatrix} - \frac{15}{10} \begin{bmatrix} 2 \\ -1 \\ 1 \\ 2 \end{bmatrix}
\]

\[
= \begin{bmatrix} 0 \\ -1/2 \\ -3/2 \\ 1 \end{bmatrix}
\]

Then the set \( \{v_1, v_2\} \) is an orthogonal basis for \( W \).

(9) We first need to find a basis for \( \text{col}(A) \) by row-reducing \( A \). We have

\[
A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}.
\]

Thus, every original column of \( A \) is a basis vector for \( \text{col}(A) \). More precisely, we let

\[
x_1 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \quad x_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad x_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}.
\]

Then \( B = \{x_1, x_2, x_3\} \) is a basis for \( \text{col}(A) \).

We now apply the Gram-Schmidt process on the vectors in \( B \) to obtain an orthogonal basis for \( \text{col}(A) \).
Let \( v_1 = x_1 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \). Then let

\[
v_2 = x_2 - \left( \frac{v_1 \cdot x_2}{v_1 \cdot v_1} \right) v_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} - \frac{1}{2} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1/2 \\ 1/2 \end{bmatrix}
\]

and

\[
v_3 = x_3 - \left( \frac{v_1 \cdot x_3}{v_1 \cdot v_1} \right) v_1 - \left( \frac{v_2 \cdot x_3}{v_2 \cdot v_2} \right) v_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} - \frac{1}{2} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} - \frac{1/2}{3/2} \begin{bmatrix} 1 \\ 1/2 \\ -1/2 \end{bmatrix} = \begin{bmatrix} 2/3 \\ 2/3 \\ -2/3 \end{bmatrix}
\]

The set \( \{v_1, v_2, v_3\} \) is an orthogonal basis for \( \text{col}(A) \).