Rounding Issues – Clarification
Math 314–006
Applied Mini-Project #1

Some explanation may be helpful for #2 (a) on page 86. Obtaining the “solution” \( x = -1.00, y = 1.01 \) depends on how one rounds to three significant digits while carrying out the involved calculations.

To solve the system

\[
\begin{align*}
0.400x + 99.6y &= 100 \\
75.3x - 45.3y &= 30.0
\end{align*}
\]

we row-reduce the associated augmented matrix:

\[
\begin{bmatrix}
0.400 & 99.6 & | & 100 \\
75.3 & -45.3 & | & 30.0
\end{bmatrix}
\overset{R_2 \rightarrow R_2 - (75.3/0.400) R_1}{\longrightarrow}
\begin{bmatrix}
0.400 & 99.6 & | & 100 \\
0 & -18700 & | & -18800
\end{bmatrix}
\]

Observe that, rounding to three significant digits,

\[
\frac{75.3}{0.400}(99.6) = 18749.7 \approx 18700
\]

and

\[
\frac{75.3}{0.400}(100) = 18825 \approx 18800.
\]

So, the numbers -18700 and -18800 in the second row of the reduced matrix are a result of the following rounding (to three significant digits):

\[
-45.3 - \frac{75.3}{0.400}(99.6) \approx -45.3 - 18700 = -18745.3 \approx -18700
\]

and

\[
30.0 - \frac{75.3}{0.400}(100) \approx 30.0 - 18800 = -18770 \approx -18800.
\]

Therefore,

\[-18700y = -18800 \implies y = 1.00534759358 \approx 1.01.\]

We now use \( y = 1.01 \) to find \( x \). We have

\[
0.400x + (99.6)(1.01) = 100.
\]

So,

\[
x = \frac{100}{0.400} - \frac{(99.6)(1.01)}{0.400} = 250 - 251.49 \approx 250 - 251 = -1.00
\]

(here we rounded 251.49 to three significant digits).