

Topics Typically Covered in 842-843

- Here is a list of qualifying exam topics drawn from a two-semester sequence taught from the second edition of J. David Logan's book *Applied Mathematics*.
- This is *not* meant to be an 842-843 syllabus nor an exhaustive list of qualifying exam topics. These are only the core topics to be studied for the exam. Additional material may be included.

1. Dimensional Analysis and Scaling

- a. The Buckingham Pi theorem
- b. Scaling

2. Perturbation Methods

- a. Regular perturbation
- b. Singular perturbation; boundary layer analysis
- c. Asymptotic expansions of integrals

3. Calculus of Variations

- a. Derivatives of functionals
- b. The Euler equation
- c. Generalizations; several functions, natural boundary conditions
- d. Mechanics: The Lagrangian and Hamiltonian formulations

4. Integral Equations

- a. Volterra integral equations
- b. Fredholm integral equations with separable kernels
- c. Symmetric kernels

5. Second-order, linear, two-point boundary value problems

- a. Fundamental solutions second-order, linear differential operators
- b. Green's functions for two-point boundary value problems
- c. The self-adjoint case; eigenfunction expansions

6. Introduction to partial differential equations

- a. Linearity versus nonlinearity; superposition
- b. Derivation of PDEs; conservation laws in \mathbf{R}^n , constitutive equations
- c. The heat equation; initial and initial-boundary value problems in n space dimensions
- d. Laplace's and Poisson's equations; boundary value problems and integral identities in n space dimensions
- e. Solution by separation of variables
- f. The Laplace transform
- g. The Fourier transform on \mathbf{R}^n

7. Wave propagation

- a. Linear and nonlinear waves; characteristics
- b. First-order, quasilinear conservation laws in one space dimension
- c. Shock formation; jump condition
- d. The wave equation; initial and initial-boundary value problems, D'Alembert's solution in one space dimension

8. Continuum mechanics in $n \leq 3$ space dimensions

- a. Kinematics
- b. Conservation of mass and momentum
- c. Application to fluid dynamics; the Navier-Stokes equations

9. Stability and Bifurcation

- a. One-dimensional problems; stability, classification of bifurcation points, exchange of stability
- b. Two-dimensional problems; the phase plane, linear and nonlinear systems, bifurcation