Math 208
Cylindrical and spherical coordinates problems

Set up and evaluate problems 1-5 in either cylindrical or spherical coordinates, whichever is more appropriate:

1. $\int_Q x\,dV$, where Q is the region with $x \geq 0$, inside the sphere $x^2 + y^2 + z^2 = 16$, and below the cone $z = \sqrt{x^2 + y^2}$.

2. $\int_0^2 \int_{-\sqrt{2x-x^2}}^{\sqrt{2x-x^2}} \int_{-2\sqrt{x^2+y^2}}^{2\sqrt{x^2+y^2}} \frac{x}{x^2+y^2}\,dz\,dy\,dx$

3. $\int_0^2 \int_0^{\sqrt{8-2y^2}} \int_y^{\sqrt{8-y^2-z^2}} \frac{1}{\sqrt{x^2+y^2+z^2}}\,dx\,dz\,dy$

4. $\int_{-2}^0 \int_{2y^2}^{-4y} \int_{-y}^{\sqrt{z-y^2}} \frac{1}{x^2+y^2}\,dx\,dz\,dy$

5. Find the mass and *z*-coordinate of the center of mass of the object inside the sphere $x^2 + y^2 + z^2 = 4z$ and below the cone $z = \sqrt{3x^2 + 3y^2}$, if the density is $\delta(x, y, z) = \frac{1}{x^2+y^2+z^2}$.

6. a) Let D be the region <u>below</u> the cone $z = \sqrt{x^2 + y^2}$ and inside the sphere $x^2 + y^2 + z^2 = 4z$ (note this is the sphere of radius 2 and center at (0,0,2), with equivalent equation $x^2 + y^2 + (z-2)^2 = 4$, or $z = 2 \pm \sqrt{4 - x^2 - y^2}$). Set up but do not evaluate $\iiint_D (x^2 + y^2)\,dV$ as a triple integral in each of rectangular, cylindrical, and spherical coordinates. <u>DO NOT EVALUATE.</u>

   b) If the sphere stays the same but the cone is replaced by the cone $z = \sqrt{3x^2 + 3y^2}$, which of these coordinate systems would still be usable to set this up using only one iterated integral? Set up at least one such triple integral. <u>DO NOT EVALUATE.</u>

Answers: 1) $48\pi + 32$ 2) $\frac{32}{3} = 10\frac{2}{3}$ 3) π

4) $\pi - 2$ 5) mass = 3π, $\bar{z} = 0.75$

6) a) Rectangular: $\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{2-\sqrt{4-x^2-y^2}}^{\sqrt{x^2+y^2}} (x^2+y^2)\,dz\,dy\,dx$ (z <u>must</u> be integrated first)

Cylindrical: $\int_0^{2\pi}\int_0^2\int_{2-\sqrt{4-r^2}}^{r} r^3\,dz\,dr\,d\theta$ or $\int_0^{2\pi}\int_0^2\int_z^{\sqrt{4z-z^2}} r^3\,dr\,dz\,d\theta$

Spherical: $\int_0^{2\pi}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\int_0^{4\cos(\phi)} \rho^4\sin^3(\phi)\,d\rho\,d\phi\,d\theta$

b) Cylindrical and spherical coordinates still work, but not rectangular. However, with this region in cylindricals, r <u>must</u> be integrated before z, giving

$$\int_0^{2\pi}\int_0^3\int_{\frac{z}{\sqrt{3}}}^{\sqrt{4z-z^2}} r^3\,dr\,dz\,d\theta .$$

Or doing it in sphericals, only the lower limit on ϕ changes from above (that was the cone),

and we get $\int_0^{2\pi}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\int_0^{4\cos(\phi)} \rho^4\sin^3(\phi)\,d\rho\,d\phi\,d\theta$