

Math 208 Parametrization Exercises

Parametrize each of the following curves (that includes specifying the starting and ending values of the parameter):

1. The straight line segment from $(2,3,7)$ to $(-1,4,3)$.
2. The portion of the circle $x^2 + y^2 = 25$ going counterclockwise from $(0,-5)$ to $(0,5)$.
3. The portion of the circle $x^2 + y^2 = 25$ going clockwise from $(0,5)$ to $(-5,0)$.
4. The portion of the circle $x^2 + y^2 = 8$ going counterclockwise from $(2,-2)$ to $(2,2)$.
5. The portion of the circle $(x-2)^2 + (y+4)^2 = 9$ going counterclockwise from $(5,-4)$ to $(2,-1)$.
6. The portion of the circle $(x-2)^2 + (y+4)^2 = 9$ going clockwise from $(5,-4)$ to $(2,-1)$.
7. The portion of the ellipse $x^2 + 4y^2 = 36$ going counterclockwise from $(-6,0)$ to $(6,0)$.
8. The portion of the ellipse $x^2 + 4y^2 = 36$ going clockwise from $(0,-3)$ to $(6,0)$.
9. The bottom half of the hyperbola $y^2 - x^2 = 4$ going from left to right.
10. The portion of the curve $x = y^3 - y$ going from $(0,1)$ to $(24,3)$.
11. The portion of the curve $y = 4$, $z = x^2 + x$ going from $(1,4,2)$ to $(-2,4,2)$.
12. The portion of the curve given by $y = x^3 - 3x$, $x = z^2$ going from $(1,-2,-1)$ to $(4,52,2)$.
13. The intersection of the cylinder $x^2 + y^2 = 16$ and the plane $z = -2y$ traveled once counterclockwise as seen from above, starting at $(0,-4,8)$.

Answers

Note: There are many alternative parametrizations.

1. $x = 2 - 3t, y = 3 + t, z = 7 - 4t, 0 \xrightarrow{t} 1$

2. $x = 5\cos(t), y = 5\sin(t), -\frac{\pi}{2} \xrightarrow{t} \frac{\pi}{2}$

3. $x = 5\cos(t), y = 5\sin(t), \frac{\pi}{2} \xrightarrow{t} -\pi$ or $x = 5\cos(t), y = -5\sin(t), -\frac{\pi}{2} \xrightarrow{t} \pi$

4. $x = \sqrt{8}\cos(t), y = \sqrt{8}\sin(t), -\frac{\pi}{4} \xrightarrow{t} \frac{\pi}{4}$

5. $x = 2 + 3\cos(t), y = -4 + 3\sin(t), 0 \xrightarrow{t} \frac{\pi}{2}$

6. $x = 2 + 3\cos(t), y = -4 + 3\sin(t), 0 \xrightarrow{t} -\frac{3\pi}{2}$ or
 $x = 2 + 3\cos(t), y = -4 - 3\sin(t), 0 \xrightarrow{t} \frac{3\pi}{2}$

7. $x = 6\cos(t), y = 3\sin(t), -\pi \xrightarrow{t} 0$

8. $x = 6\cos(t), y = 3\sin(t), \frac{3\pi}{2} \xrightarrow{t} 0$ or $x = 6\cos(t), y = -3\sin(t), \frac{\pi}{2} \xrightarrow{t} 2\pi$

9. $x = 2\sinh(t), y = -2\cosh(t), -\infty \xrightarrow{t} \infty$ or
 $x = 2\tan(t), y = 2\sec(t), \frac{\pi}{2} \xrightarrow{t} \frac{3\pi}{2}$

10. $x = t^3 - t, y = t, 1 \xrightarrow{t} 3$

11. $x = t, y = 4, z = t^2 + t, 1 \xrightarrow{t} -2$ or $x = -t, y = 4, z = t^2 - t, -1 \xrightarrow{t} 2$

12. $x = t^2, y = t^6 - 3t^2, z = t, -1 \xrightarrow{t} 2$

13. $x = 4\cos(t), y = 4\sin(t), z = -8\sin(t), \frac{-\pi}{2} \xrightarrow{t} \frac{3\pi}{2}$