Math 107H Exam 2

Do any five problems. Indicate on the exam sheet which ones you want graded.
Show your work. Justify your conclusions.

[10] 1. Use the formula for the derivative of the inverse to show that

\[
\frac{d}{dx} \arcsin x = \frac{1}{\sqrt{1 - x^2}},
\]

for \(-1 < x < 1\).

2. Find the indefinite integral of the given function.

[2] a. \(f(x) = \frac{e^{\sqrt{x}}}{\sqrt{x}}\), for \(x > 0\).

[3] b. \(g(t) = \arctan t\). (Hint: Integrate by parts with \(v' = 1\).)

[2] c. \(\psi(\theta) = \cos^3 \theta\).

[3] d. \(f(x) = \tanh x \ln (\cosh x)\).

[10] 3. Find the indefinite integral of \(f(x) = \frac{2x}{x^2+2x+2}\).

[10] 4. Find the indefinite integral of \(h(x) = e^x \cos x\).

[5] Let For \(x > 0\), let \(w(x) = \frac{1}{x^3+x^2}\).

[7] a. Find the partial fraction decomposition of \(w(x)\).

[3] b. Find the indefinite integral of \(w(x)\).

[10] 6. Find the indefinite integral of \(g(x) = \frac{1}{x^3\sqrt{x^2-1}}, x > 1\).

7. Is the integral convergent or divergent?

[3] a. \(\int_0^\infty \frac{1}{x+x^2} \, dx\).

[4] b. \(\int_2^\infty \frac{1}{x \ln x} \, dx\).

[3] c. \(\int_0^\infty xe^{-x} \, dx\).