Outline for Final Exam


1.5 Limits. Functions $f(x)$ for which $\lim_{x\to c} f(x)$ does not exist. Functions $f(x)$ for which $\lim_{x\to c} f(x)$ exists but is not equal to $f(c)$. One-sided limits. Calculation of limits.

1.6 Continuous and discontinuous functions.

2.1 The slope of a secant line. The derivative. The derivative as an instantaneous rate of change. The derivative as the slope of a tangent line. Computing derivatives from the definition

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}.$$ 

Finding the equation of the tangent line to a graph at a given point.

2.2 Rules for differentiation: Differentiation of constants. The power rule. Differentiation of sums, differences and constant multiples.

2.3 Average rates of change and marginals (derivatives). The derivative as an instantaneous rate of change. Marginal cost, marginal revenue, marginal profit. The demand function.

2.4 Rules for differentiation: The product and quotient rules.

2.6 Higher derivatives.

2.7 Implicit differentiation.

2.8 Related rates. Setting up and solving related rates problems.

3.1 Increasing and decreasing functions. Critical numbers and critical points. Charting the sign of the first derivative.

3.2 Relative extrema and the first derivative test. Absolute extrema. Finding the absolute extrema of a function on a closed interval $[a, b]$.

3.3 Concavity, inflection points and the second derivative test.

3.4 Optimization. Setting up and solving optimization problems. Primary and secondary equations.

3.5 Optimization problems in business. Maximization of profit and revenue. Minimization of cost and average cost.

3.6 Asymptotes of rational functions. Finding vertical and horizontal asymptotes. Limits and asymptotes.

3.7 Graphing: Domain, intercepts, asymptotes. Using the first derivative to determine critical numbers, critical points, relative extrema and intervals of increase and decrease. Using the second derivative to determine intervals over which the graph is concave up or down, and inflection points.
3.8 Using the differential $dy$ to approximate the change $\Delta y$. Marginal analysis.

4.1 Exponential functions. Exponential growth and decay. Graphs of exponential functions.

4.2 The natural exponential function $\exp(x) = e^x$. Compound and continuously compounded interest. The effective rate of interest. Present value.

4.3 Derivatives of exponential functions. The differentiation rules

$$\frac{d}{dx} e^x = e^x \text{ and } \frac{d}{dx} e^{u(x)} = u'(x)e^{u(x)}.$$

4.4 The natural logarithm function $\ln x$. Properties of the natural logarithm. Solving exponential equations. Applications to finance.

4.5 Derivatives of logarithmic functions. The differentiation rules

$$\frac{d}{dx} \ln x = \frac{1}{x} \text{ and } \frac{d}{dx} \ln u(x) = \frac{u'(x)}{u(x)}.$$

4.6 Exponential growth and decay. Radioactive decay. Applications to finance.


5.2 Integration by substitution.

5.3 Integration of exponential and logarithmic functions.

5.4 Definite integrals and the area under a curve.

5.5 Applications of the definite integral: The area between curves. The average value of a function on an interval. The consumer surplus and the producer surplus.

5.6 Numerical integration. The midpoint rule for approximation of a definite integral.