Extra Credit 2

15 October 2009

1. Is there a function that is both even and odd? If so, demonstrate its symmetries. If not, explain why it cannot exist.

   Assume that $f \in \mathbb{R}^{\mathbb{R}}$ is both even and odd. That is, f is symmetric with respect to both the origin and the y-axis. Then, for each $(x, y) \in f$, $(-x, y), (-x, -y) \in f$ also. Since f is a function, $y = -y$. Hence, $2y = 0$, forcing $y = 0$. Since this was regardless of x, $f(x) = 0$ for all $x \in \mathbb{R}$.

   Consider $f : \mathbb{R} \to \mathbb{R}$ by $f(x) := 0$. Letting $y = f(x) = 0$, observe that for $x \in \mathbb{R}$, $(-x, 0) \in f$. Thus, f is symmetric with respect to both the y-axis and the origin. Hence, it is both even and odd.

2. Is there a function that is symmetric with respect to the x-axis? If so, demonstrate its symmetries. If not, explain why it cannot exist.

   Assume $f \in \mathbb{R}^{\mathbb{R}}$ has symmetry with respect to the x-axis. Then, given any $(x, y) \in f$, $(x, -y) \in f$. Then as f is a function, $y = -y$. Hence, $2y = 0$, forcing $y = 0$. Since this was regardless of x, $f(x) = 0$ for all $x \in \mathbb{R}$.

   Consider $f : \mathbb{R} \to \mathbb{R}$ by $f(x) := 0$. Letting $y = f(x) = 0$, observe that for $x \in \mathbb{R}$, $(x, 0) \in f$. Thus, f is symmetric with respect to the x-axis.