

Extra Credit 1

17 September 2009

Consider the empty relation, $\emptyset \subset \mathbb{R}^2$.

1. Is \emptyset a function? Explain your response.

Recall that a function is a relation f such that for all first coordinates $x \in \mathbb{R}$, there is at most one $y \in \mathbb{R}$ with $(x, y) \in f$. To fail this, a relation must have points $(x, y_1), (x, y_2) \in f$ with $y_1 \neq y_2$. Since \emptyset has no points, this cannot happen. Hence, the condition is satisfied. That is, \emptyset is a function.

2. What is $\text{dom}(\emptyset)$ and $\text{ran}(\emptyset)$?

Recall that

$$\text{dom}(\emptyset) = \{x \in \mathbb{R} : \text{there is } y \in \mathbb{R} \text{ with } (x, y) \in \emptyset\}$$

and

$$\text{ran}(\emptyset) = \{y \in \mathbb{R} : \text{there is } x \in \mathbb{R} \text{ with } (x, y) \in \emptyset\}.$$

Since there are no points in \emptyset , both of these are empty. Explicitly,

$$\text{dom}(\emptyset) = \text{ran}(\emptyset) = \emptyset.$$