1. (16 pts) For each of the below either give an example or state that there exist no such examples:

(a) A relation, described as a set of ordered pairs \((x, y)\), that does not define \(y\) as a function of \(x\).

For example, \(\{(0, 0), (0, 1)\}\)

(b) A trinomial in \(x\) that is prime.

For example, \(x^2 + x + 1\).

(c) A polynomial with greatest common factor \(-2a\).

For example, \(-2ax - 2a\).

(d) A number \(x\) so that \(x^0 = 0\).

No such examples, since \(x^0 = 1\) for all \(x\), except that \(0^0\) is undefined.

(e) A system of linear equations with no solution.

For example, a system that represents parallel lines:

\[
\begin{align*}
y &= x \\
y &= x + 1
\end{align*}
\]

(f) A system of linear equations with exactly two solutions.

No such examples. It is impossible for any two lines to cross in exactly two places.

(g) A system of linear equations with an infinite number of solutions.

For example, a system that represents the same line:

\[
\begin{align*}
y &= x \\
2y &= 2x
\end{align*}
\]

(h) A trinomial with the following factors: \(2, x + 4, x - 4\).

For example, \(x^2 + x^2 - 32\).
2. (15 pts) Do one of the following two problems. If you do both problems, I will give you credit for the better of the two.

(a) A car and a bus set out at 2pm towards Ogallala on I-80. The average speed of the car is 30mph slower than twice the speed of the bus. At 4pm, both the car and the bus are pulled over by state troopers, but the car is 100 miles farther than the bus. What is the average speed of the car?

Let \( c \) be the average speed of the car and \( b \) be that of the bus. Then \( c = 2b - 30 \). In general, \( d = rt \), so if \( d \) is the distance the bus travelled then \( d + 100 \) is the distance the car travelled. For both the car and the bus, the time \( t = 2 \) hrs. Now for the bus, we have \( d = b \cdot 2 \) and for the car, we have \( d + 100 = c \cdot 2 \). Substituting for \( c \), we get \( d + 100 = (2b - 30) \cdot 2 \), and substituting for \( d \), we get \( 2b + 100 = 4b - 60 \). Solving, we get \( b = 80 \), so \( c = 2b - 30 = 160 - 30 = 130 \). The average speed of the car is 130mph.

(b) You are working in a chemistry lab that is stocked with 10% acid solution and 30% acid solution. However, you need 10 liters of 15% acid solution. How much of each solution must you mix in order to get the solution you need?

Let \( x \) and \( y \) be the amounts in liters that we will mix of the 10% and 30% solutions, respectively. Then \( x + y = 10 \) and \((.10)x + (.30)y = (.15)10\). The first equation gives \( x = 10 - y \) and the second simplifies to \( x + 3y = 15 \). Substituting for \( x \), we get \( 10 - y + 3y = 15 \), which gives \( 2y = 5 \) so \( y = 2.5 \). But now \( x = 10 - y = 10 - 2.5 = 7.5 \), so we must mix 2.5L of 10% solution with 7.5L of 30% solution.

3. (5 pts) Let’s say that \( x \) is related to \( y \) if \( y = 2x + 1 \). Does this relation define \( y \) as a function of \( x \)? Justify your answer.

Yes since given an \( x \) value you can say exactly to which \( y \) value it corresponds.

Identify the dependent and independent variables:

\( y \) depends on \( x \), so \( y \) is the dependent variable, and \( x \) the independent variable.

4. (15 pts) Multiply:

(a) \((2m - 5)(3m^2 + 4m - 5)\)

\[ 6m^3 - 7m^2 - 30m + 25 \]

(b) \((2k + q)^2\)

\[ 4k^2 + 4kq + q^2 \]

(c) \((6x + y)(6x - y)\)

\[ 36x^2 - y^2 \]
5. (11 pts) Let \( f(x) = -2x^2 + 5x - 6 \) and let \( g(x) = 7x - 3 \). Evaluate each of the following:

(a) \((f + g)(x)\)

\[-2x^2 + 12x - 9\]

(b) \((f - g)(x)\)

\[-2x^2 - 2x - 3\]

(c) \((f + g)(-2)\)

\[-41\]

6. (10 pts) Fully simplify each expression, writing each with no negative exponents:

(a) \((-2x^4y^{-3})^0 (-4x^{-3}y^{-8})^2\)

\[\frac{16}{x^6y^{16}}\]

(b) \(\left(\frac{4p^2}{q^4}\right)^3 \left(\frac{6p^8}{q^{-8}}\right)^{-2}\)

\[\frac{16}{9p^{10}q^{28}}\]

7. (20 pts) Factor:

(a) \(ak^3 + 2ak^2 - 9ak - 18a\)

\[a(k + 3)(k - 3)(k + 2)\]

(b) \(18k^2 - 200j^2\)

\[2(3k + 10j)(3k - 10j)\]

(c) \(a^3u^3 + c^3\) (Remember that, in general, \(x^3 + y^3 = (x + y)(x^2 - xy + y^2)\)).

\[(au + c)(a^2u^2 - auc + c^2)\]

(d) \(4p^2 + 3p - 1\)

\[(4p - 1)(4p + 1)\]

(e) \(2(1 - x)^4 + 2(1 - x)^3 - (1 - x)^2\)

\[(1 - x)^2(2x^2 - 6x + 3)\]
8. (8 pts) Replace $x$ with $-1$ and $y$ with $2$ to give an example showing that $(x+y)^2$ does not equal $x^2 + y^2$.

\[
(-1 + 2)^2 = 1
\]

and

\[
(-1)^2 + 2^2 = 5
\]

While it is not true in general that $(x+y)^2 = x^2 + y^2$, the equation is true for certain values of $x$ and $y$. Give an example of choices for $x$ and $y$ that make the equality true: $(x+y)^2 = x^2 + y^2$.

For example, $x = y = 0$.

9. Bonus (2 pts) Describe all choices of $x$ and $y$ that make the equation true: $(x+y)^2 = x^2 + y^2$.

All choices of $x$ and $y$ work provided that at least one of them is zero.

Bonus (2 pts) Prove that you have in fact described all such choices (that is, give an argument to convince me that you haven’t forgotten any).

For example, $(x+y)^2 = x^2 + y^2$ if and only if $x^2 + 2xy + y^2 = x^2 + y^2$ if and only if $2xy = 0$ if and only if $x = 0$ or $y = 0$. 