Practice Quiz 5 Solutions

**Instructions**: Answer each question, and when required explain your answer. Your explanation must be clear and complete. You may refer to your book, your notes and your homework papers. You may use a calculator.

[1] (10 points) Suppose a screening test for a certain virus is positive 95% of the time for infected people and 2% of the time for uninfected people. Also suppose that 1% of the population is infected.

(a) Draw a probability tree diagram for the experiment that consists of randomly picking a person, who may be either infected or healthy, and then giving the person the test. *Done in class.*

(b) What is the probability that the person is infected? \( A: \frac{99}{100} \)

(c) What is the probability that the person tests positive given that the person is not infected? \( A: \frac{198}{9900} \)

(d) What is the probability that the person is not infected given that the test is positive? \( A: \frac{198}{293} \)

(e) What is the probability that the test gives correct results? \( A: \frac{9797}{10000} \)


(a) Find (i) and (ii), the missing probabilities, in the tree diagram. \( A: \) (i) is \( \frac{1}{5} \) and (ii) is \( \frac{3}{8} \)

(b) Find \( P(R \cap P) \), \( P(S \cap P) \), and \( P(P) \). \( A: \frac{3}{10}, 1/2, \) and \( \frac{4}{5} \), respectively.

(c) Find \( P(R \cap Q) \), \( P(S \cap Q) \), and \( P(P) \). \( A: \frac{1}{15}, 2/15, \) and \( \frac{11}{30} \), respectively.

(d) Find and interpret \( P(S \mid P) \). \( A: \frac{5}{8} \). This is the probability of \( S \) given that \( P \) happens.

(e) Find and interpret \( P(Q \mid R) \). \( A: \frac{2}{11} \). This is the probability of \( Q \) given that \( R \) happens.

[3] (6 points) Which is the best deal over a 3-year period: investing at 7.2% compounded annually, investing at 7% compounded monthly or investing at 6.8% compounded daily? Show how you determine the answer.

*Note that the 3-year period doesn’t matter. If one outgrows the others after 1 year, then it will continue in the later years. It’s enough to see how $1 grows over 1 year under the different circumstances. The best deal is 6.8% compounded daily.*

[4] (7 points) How much money would have to be invested in an account at 4.25% annual interest to achieve a balance of $50,000 in 20 years if

(a) the account pays simple interest?

(b) the account compounds interest semiannually?

*In both cases, we’re given the future value, \( F \), and are asked to find the principal. In part (a), the formula for future value is \( F = P(1+rt) \), so we can plug things in and find that \( P = \$27,027.03 \), and in part (b), the formula for future value is \( F = P(1+r/m)^m \), so we can plug things in and find that \( P = \$21,561.91 \).*
(7 points) (a) Use the monthly payment formula to determine the monthly payment for a 60-month amortized loan of $25,495 at 4.5% interest.

(b) Use an amortization table to find the monthly payment for the loan and compare the result with the monthly payment found in part (a).

The answer is the same in both cases, but the work is not. In part (a), use the formula mentioned to find the monthly payment of $475.30. In part (b), the relevant entry in the amortization table is $18.643019/$1000 loaned. The loan is for 25.495 thousand, so multiplying (25.495)(18.643019) gives the answer.