Research Statement
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My research in algebraic coding theory thus far has had three major components, which I will summarize below. In addition to my research in coding theory, I have a strong interest in the mathematical education of teachers, and I will describe my plans for research in this area in the final section of this document.

Introduction to Coding Theory

Reliable transmission of digital data is essential in today's society given its role in technological advances such as transmission of satellite images, wireless communications, and data storage/retrieval. The aim of *channel coding* is to allow transmission of information over a noisy *channel* in such a way that the receiver can recover the original information. To accomplish this, some level of redundancy is added to the information before transmission; this redundancy is added in such a way that it can be used to correct for the noise introduced by the channel. As a classic example of channel coding, we consider the case of compact discs. When storing music or data on a CD, one wants to ensure that the information can be retrieved even in the presence of scratches. Thus, it is necessary to include some redundancy with the information to guarantee that it is possible to recover the information in spite of the noise produced by these scratches. While the additional redundancy provides greater error-correction capabilities, it detracts from the amount of information that can be stored on the CD. Thus, a primary goal of coding theory is to design codes that minimize the amount of redundancy necessary to provide a certain level of error correction. Another focus of modern coding theory is to find decoding algorithms that will enable the user to efficiently recover the information from the erroneous data received.

My research in algebraic coding theory thus far has consisted of three main projects. The first project has focused on understanding the particular class of efficient decoding algorithms known as *iterative message-passing decoders*. Specifically, the performance of these decoders is significantly impaired by the presence of *pseudocodewords*, and so we focused on investigating the structural properties of these error patterns. The second project has centered around the neuroscience model for how the brain "decodes" what stimulus it has encountered given a noisy neural response to that stimulus, a process known as *neural coding*. Therefore, we focused on understanding the potential links or parallels of that model to the coding theory model for communication in the presence of noise, with an eye toward using key coding theory principles to help explain some "paradoxical" neural coding phenomena. This project is farther afield from the typical coding theory topics, but involves an interesting application of the coding theory paradigm. Finally, I have also investigated the design of "good" codes for a new application of channel coding, namely the provision of error correction for random linear network coding. In particular, I have examined the connections between rank-metric and matrix codes as well as properties of these codes that determine their effectiveness for providing error correction in the network coding context. The first two research areas were the focus of two collaborative interdisciplinary research projects, and I will describe those briefly below. The last project has formed the bulk of my thesis work, and I will describe that in greater detail, together with some future work I have planned in that direction.

Pseudocodewords of iterative message-passing decoders

Shannon [1] proved that by appropriately adding some redundancy to the data, a process known as *encoding*, it is possible to efficiently and reliably transmit data across virtually any channel. His proof was probabilistic, however, and gave little insight into how to construct these codes or efficiently decode them. One major breakthrough in finding good codes with efficient decoders came from *codes on graphs*, such as low-density parity check codes, and iterative message-passing decoders [2] – [4]. While iterative message-passing decoders are extremely efficient and easy to implement, they are suboptimal in that there are certain error patterns that are theoretically correctable, but are not correctable with these decoders. Thus, much research has been done to identify and characterize the source of these decoding failures with the hope of improving the decoding algorithms and/or the codes to which these are applied [5] – [10].

Wiberg's thesis [6] established that iterative message-passing decoding algorithms are precisely modeled by computation trees, and thus *computation tree pseudocodewords* are the root of these decoding failures; however, analysis of computation tree pseudocodewords has been largely intractable thus far. Meanwhile, there is an intuitive connection between *graph covers* and iterative message passing decoders, and thus research on *graph cover pseudocodewords* may prove applicable in this area. These objects have been heavily studied because, in addition to the intuitive link to iterative message passing decoders, they have been shown to be the exact source of errors for *linear programming decoding* [10]. As a result, there has been much fruitful research on the structure and form of graph cover pseudocodewords and a related theoretical decoding algorithm known as *graph cover decoding* [7] –[10]. Together with Eric Psota and Dr. Lance Pérez in the Electrical Engineering Department and Deanna Dreher, Nathan Axvig, and Dr. Judy Walker in the Mathematics Department at UNL, I sought to bridge the gap between these two areas by examining *universal covers*, which subsume both computation trees and graph covers. Specifically, we analyzed the existence of *universal cover pseudocodewords* and their relationships to computation tree and graph cover pseudocodewords as a means for translating the results on graph covers to the case of computation trees [11, 12]. We characterized some sufficient conditions under which a graph cover pseudocodeword can be realized on a *connected* cover, a necessary condition for it to give rise to a computation tree pseudocodeword. We also characterized when both types of pseudocodewords can be derived from universal cover pseudocodewords, and we proposed a theoretical decoding algorithm on the universal cover that would reduce to the graph cover decoding algorithm while maintaining an explicit link to computation tree pseudocodewords and the original iterative message-passing decoding algorithm [12].

Coding theory perspective on neural codes

Neural coding aims to understand how information is reflected in the brain's response to a given stimulus. It has been experimentally observed that the pattern of neural activity is highly correlated with different features of the encountered stimulus. For example, experimental data has shown that neurons in the V1 area of the visual cortex are tuned to the orientation of objects in the visual field: Each neuron in V1 appears to have a preferred angle of orientation to which it responds most strongly and the neuron's firing rate tends to decay as the distance from that preferred angle increases [13] - [15]. Thus, neuroscientists view the brain as *encoding* different features of the stimulus in its neural response. From experimental data, it appears that this encoding process is typically noisy in that the same stimulus will induce a variety of slightly different neural responses; therefore, neuroscientists aim to understand how the brain uses these noisy responses to determine, or *decode*, what stimulus was encountered. Different types of *neural codes* are studied by neuroscientists in different contexts; depending on what aspects of the neural activity are deemed important, different types of codes are considered, such as *temporal, rate*, or *combinatorial* codes.

Together with mathematical neuroscientists Drs. Carina Curto and Vladimir Itskov, and coding theorists Zach Roth and Dr. Judy Walker, I have focused on properties of combinatorial codes. A *combinatorial code* is a subset of $\{0,1\}^n$, with each codeword corresponding to the ideal response to a stimulus, so that the i^{th} coordinate of the codeword is 1 if and only if the i^{th} neuron is typically expected to fire in response to that stimulus. Thus, we view the neural activity as a binary code whose properties can be examined from a coding theoretic perspective. In fact, both neural coding and mathematical coding theory share their roots in Shannon's seminal paper [1], but they have subsequently diverged greatly with mathematical coding theorists developing a complementary perspective on the nature and function of codes that is unfamiliar to most neuroscientists. In particular, there are a number of characteristics of combinatorial codes that are viewed as "paradoxes" in the neural coding world, but that seem natural from a mathematical coding theory perspective. For example, neuroscientists often ask "Why are not all neural patterns observed?" and "Why is there so much coding redundancy?" [16] since these appear to be deficiencies in the neural code, but mathematical coding theorists view these as valuable features of the code that promote greater error correction. Thus, our group has aimed to provide a mathematical coding theory perspective in the analysis of certain families of combinatorial neural codes, namely *1-dimensional* and *2-dimensional receptive field* (RF) codes. Our main goal has been to attempt to explain why the brain might use these codes.

To analyze the optimality of these codes, we have measured their error-correcting capabilities on a *binary asymmetric channel* that reflects the biological constraints that make it more likely for a neuron to fail to fire than to spontaneously fire. As a baseline, we compared the performance of the RF codes under *maximum likelihood* (ML) decoding to the performance of random codes with matched parameters. At first glance, the RF codes appear horribly sub-optimal because their probability of correct decoding is drastically lower than that of the random codes. However, we have proposed a new notion of error-correction with an error-tolerance with respect to a natural metric on the stimulus space, and under these conditions the RF codes quickly "catch up", i.e. their performance matches or exceeds the performance of the random codes for relatively small error tolerances. To explain this "catch up" phenomenon, we have proposed a notion of *ML similarity*, and we have shown that this similarity measure is highly correlated with the induced stimulus-space metric on the RF codes, but not with the induced metric on the random codes, thus explaining why the error tolerance induced by the stimulus-space metric drastically improves the performance of the RF codes, but not the random codes. Thus, with respect to these biological conditions, we have shown why the brain might employ RF codes even though they initially appear to have such poor error-correcting capabilities [17].

Dissertation Work: Rank-metric and matrix codes

The notion of network coding was first proposed by Ahlswede et al. in 2000 [18] and has since garnered significant attention because of its many important applications, such as cellular networks, sensor networks, and peer-to-peer (P2P) networks. Before the introduction of network coding, the primary method for passing information within a network was routing, in which each network node may only replicate or split the data it receives and then send out that data toward the intended receiver, or sink. Network coding provides an alternative approach where each node has the ability to intelligently combine, or *code*, the information coming into it, and then pass along this new encoded data toward the sink; this coding often enables the network to achieve a higher *throughput*, i.e. a higher amount of information transmitted from the source to each receiver in a single unit of time. Network coding first proved valuable in cases where there was a fixed network structure that inherently had some bottlenecks that limit the amount of routed information that can be sent in a given transmission through the network. As an example of the network coding gain, we consider the fixed network structure (i.e. prescribed set of nodes and links) in Figure 1, and then analyze the network coding scheme shown with it. Within the constraints of this network, we desire to transmit two binary vectors, or *packets*, of information $\mathbf{b}_1$ and $\mathbf{b}_2$ from the source v_1 to the receivers v_6 and v_7. With a simple encoding scheme at v_4, namely binary addition, both sink nodes v_6 and v_7 receive $\mathbf{b}_1 \oplus \mathbf{b}_2$ from v_4 via v_5. Then using the other vector they received, v_6 and v_7 are

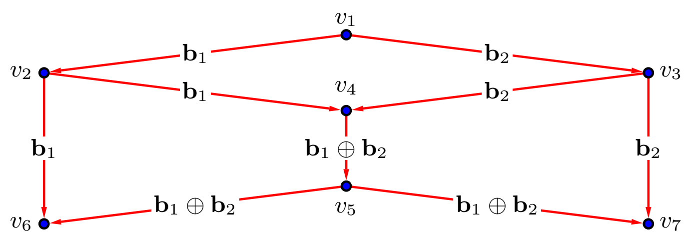

FIGURE 1. Network configuration with a simple network coding scheme to achieve capacity.

able to decode, again via binary addition, to receive both $\mathbf{b}_1$ and $\mathbf{b}_2$ within a single transmission. If only routing were used, then v_4 would have to send $\mathbf{b}_1$ to v_5 and on to v_7 in one transmission and then send $\mathbf{b}_2$ to v_5 and on to v_6 in a second transmission in order for v_6 and v_7 to receive both packets of information. Thus, network coding significantly improves the throughput in this instance. For networks with a single source and multiple receivers, as in the previous example, Ahlswede et al. [18] prove an analogue to the Max-Flow Min-Cut Theorem that gives a value for the maximum information flow of the network per unit of time, and they prove the existence of network codes that can achieve this information rate. This work is similar to that of Shannon's channel coding theorem in that it probabilistically proves the existence of good codes, but does not actually give a construction. Li, Yeung, and Cai [19] introduce the notion of *linear network codes*, which are codes where all operations at intermediate nodes are required to be linear over some finite field. They go on to show that, for these same types of networks, it is actually possible to achieve the network capacity with linear codes alone. Koetter and Médard [20] prove that it is in fact sufficient to use random linear network coding to achieve capacity; in other words, it is sufficient to simply allow each internal node to pass along a randomly generated linear combination of its inputs as long as the coefficients of each linear combination live over a sufficiently large finite field. As a result, random linear network coding has become widely implemented in many network settings.

A significant drawback of network coding arises, however, when noise is introduced at any of the internal nodes or links. Even a single error introduced somewhere early in the network can propagate through to potentially corrupt all the final outputs; thus, some form of error correction is necessary. Since random linear network coding outputs linear combinations of the input vectors, the subspace of input vectors is preserved at the output. Kötter and Kschischang [21] propose the use of *subspace codes*, i.e. carefully chosen collections of subspaces, to provide error correction for network coding. They also propose a simple construction for subspace codes via the lifting of linear codes whose codewords are either matrices over $\mathbb{F}_q$, known as *matrix codes*, array codes, or space-time codes over a finite field, or whose codewords are vectors over $\mathbb{F}_{q^m}$ equipped with the rank distance, known as *rank-metric codes*; specifically they propose lifting Gabidulin codes, which comprise a class of optimal rank-metric codes. Additionally, they introduce a metric on the collection of subspaces and define a minimum-distance decoder for subspace codes. The subspace metric turns out to be a scalar multiple of the rank metric when the subspaces are lifted from matrix or rank-metric codes [22], and so it is valuable to study the distance properties of such codes.

The question that has driven much of my research thus far is: how does one design matrix or rank-metric codes that are both efficient, i.e. have high dimension, and effective at error correction, i.e. have high minimum distance. Analogues of the MacWilliams Identities for block codes have been developed for rank-metric [23] and matrix codes [24], demonstrating that the inherent trade-off between dimension and minimum distance for a code is reversed for its dual code; specifically, if a code has high dimension and low minimum distance, then its dual code will have low dimension and high minimum distance. Thus, with an aim towards finding codes with a perfectly balanced and optimal trade-off, I have focused my study on self-dual matrix codes; in particular, I have worked to enumerate all inequivalent self-dual matrix codes of relatively short length over small finite fields. In general, two codes are considered equivalent if there is a semi-linear map between them that preserves the distance distribution. Given the focus on self-dual matrix codes, an additional property is required, namely that an equivalence map send a self-dual code to another self-dual code. Towards this end, I have characterized the subset of matrix-equivalence maps that commute with the dual and thus maintain the property of self-duality. Building off an approach proposed in [25] to enumerate inequivalent self-dual block codes using double-cosets, I have enumerated inequivalent self-dual matrix codes of particularly small lengths over small finite fields. I have extended this approach further to enumerate inequivalent self-dual matrix codes of longer lengths and larger field sizes by exploiting the existing enumeration of inequivalent self-dual block codes in the literature. As a further application of this work in duality theory, I have used the analogues of the MacWilliams Identities to show that a family of subspace codes known as *spread codes* [26] cannot be obtained by lifting any matrix code, even a nonlinear code, thereby proving that these codes have better parameters than any lifted codes.

I have also addressed the more general question of describing the class of all matrix-equivalence maps, including those that do not preserve self-duality. I have characterized the collection of matrix-equivalence maps, i.e. $\mathbb{F}_q$-semilinear maps that preserve rank weight, and contrasted this with the class of rank-metric-equivalence maps given in [27]. The subset of rank-metric/matrix-equivalence maps that fix a given code is known as the *rank-metric/matrix-automorphism group* of that code. I have characterized the matrix-automorphism group of the matrix codes formed by expanding Gabidulin codes. This work extends work begun by Berger in [27], where he described the rank-metric-automorphism group of Gabidluin codes. Unfortunately, there was an flaw in his proof; after finding counter-examples to his characterization, I have proved a corrected statement of his theorem.

Future Work: Applications of Rank-Metric and Matrix Codes

I am beginning to work on applications of these equivalence maps to public-key cryptography. As mentioned previously, Gabidulin codes are widely used for generating subspace codes, but they have also found applications in defining a public-key cryptosystem, known as the GPT cryptosystem, analogous to the McEliece cryptosystem [28]. In this setting, Gabidulin codes have proven valuable because they have high minimum distance and an efficient decoding algorithm, but are resistant to combinatorial decoding attacks by cryptanalysts when the code in use is unknown. One drawback of these codes, however, is their highly structured nature because it enables cryptanalysts to recover the original code and crack the cryptosystem. To attempt to disguise the structure of the code, a simple rank-metric-equivalence map, namely a permutation matrix over the base field, is employed in one updated version of the GPT cryptosystem; however, the permutation matrix still does not provide sufficient protection to resist Overbeck's attack [28]. To circumvent this attack, Gabidulin proposed using a permutation matrix over an extension field, which no longer guarantees that the modified code will be equivalent to the original Gabidulin code, and thus the high minimum-distance property may be lost. As a possible alternative to this, I am investigating the use of a different class of equivalence maps, specifically those that arise from matrix-equivalence maps, to further disguise the structure of Gabidulin codes, while still maintaining the distance distribution.

Mathematical Education of Teachers

As I mentioned above, I have become extremely interested in the mathematical education of teachers over the past four years. To date, I have not produced any research in this area, but I have assisted with a number of graduate courses through the *NebraskaMATH* grant and the *Nebraska Math and Science*

Summer Institutes, namely Functions, Algebra and Geometry for K-3 Mathematics Specialists, Matrix Algebra for Teachers, and Number Theory and Cryptology. To further my exposure to the research side of things, I have begun participating in the bi-weekly meetings for the research component of NebraskaMATH, and last semester I took the education course "Mathematical Ways of Knowing and the Art of Teaching". In the following, I will describe some of the types of research projects I am interested in pursuing regarding the mathematical education of teachers.

While I have enjoyed my experiences teaching math content courses for teachers, and have found success in that area, I believe that it is also important for me to contribute to the knowledge base on effective practices in the mathematical education of teachers. Specifically, I plan to be involved in the creation of professional development programs for teachers, and the evaluation of the impact of various aspects of these programs to help determine ways to effectively and efficiently reach large populations of students.

The NebraskaMATH grant, with which I have had significant experience, has focused its efforts on providing professional development for in-service teachers to enable them to offer mathematically-challenging courses while accommodating the individual cognitive needs of their students, particularly at-risk populations. One major initiative within this grant is the *Primarily Math* program, which has aimed at helping in-service teachers to become K-3 math specialists. This audience has been targeted because of research identifying early childhood math education as a key factor in U.S. children's weak mathematical performance, particularly for poor and minority children [29]. Furthermore, this program has focused on easing teachers' transitions into specialized roles as math coaches or math-intensive teachers because of previous research demonstrating the success of elementary math specialists in addressing these issues [30], [31]. This program utilizes separate math content, pedagogy, and developmental psychology courses to simultaneously strengthen the teachers' math content knowledge and habits of mind while addressing the relevant cognitive development of K-3 children to inform effective pedagogy in various K-3 settings, ranging from math-focused lessons to child-led exploration to math learning-focused school-home relationships.

The research component of Primarily Math aims to answer three main questions: 1) To what extent do the attitudes, knowledge, and habits of mind emphasized in the program result in changes in the teachers' practices; 2) How does the introduction of new leadership roles affect the mathematical culture of the schools; 3) What is the impact of these teachers' experiences on the mathematical beliefs and skills of the children in the school, i.e. are there corresponding gains for the students. To address these questions, the program has implemented a mixed methods design involving both survey and test data for the teachers and students together with individual follow-up interviews with selected teachers and students.

My exposure to this program has piqued my interest in the question of how to effectively prepare and support elementary teachers. I am particularly interested in some of the design principles that have enabled these teachers to teach mathematics in a way that emphasizes both mathematical content knowledge and student feelings of self-efficacy. Additionally, I am interested in how to support teachers who have deeper knowledge of mathematical content and pedagogy to have impacts on a larger scale, beyond their individual classrooms. While the research design of this program is much more ambitious than I would initially undertake, it has shaped my perspective on the key aspects of measuring the effectiveness of professional development for teachers and has informed the type of work I hope to pursue in collaboration with mathematics educators and local school districts. Specifically, I am interested in researching the impact of various professional development programs on teacher knowledge, teacher attitudes, and finally the impact on important student outcomes such as achievement, beliefs about mathematics, and their interest in pursuing further study in related fields. One essential component of the research design is classroom observations to ensure that the courses are actually impacting the teachers' practices since enhancing teacher knowledge of content and pedagogy in and of itself is of no use unless it actually translates into the classroom. Additionally, a focus on a variety of measures of student success is valuable for ensuring that the students are benefitting from the teachers' gains. These measures can help to inform future modifications to the professional development program to guarantee that significant populations of students are actually being helped as a result of these programs.

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