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5/3/11 Math Ed 949

Integrating a Disciplinary View of Mathematics Into the Classroom

Knowing Mathematics

Sense-making

To know something in mathematics is not simply to be able to regurgitate facts or fluently perform a procedure. At the heart of the discipline is the notion that there is some underlying logic tying together all of these facts or skills and that, fundamentally, everything in mathematics should make sense. In particular, it should be possible to justify all of one's arguments and conclusions. This ability to justify is one of the essential components of knowing something in mathematics, and its importance is reflected in the expectation of absolute proof:

> A classical mathematical proof ... begin[s] with a series of axioms … Then by arguing logically, step-by-step, it is possible to arrive at a conclusion. If the axioms are correct and the logic is flawless, then the conclusion will be undeniable. (Singh, 1997, p. 21)

This is the desired standard of proof so that the conclusions may serve as the foundation for generating more knowledge. In particular, once a claim has been established as fact by way of proof, it is possible to use it to motivate and answer additional questions thereby advancing one's knowledge further. Thus, it is essential that the foundational claims be verified by sound logic.

Intuition

Although the desired final product is sound logical justification, that is not always the route followed in the course of developing that knowledge. Bruner and King and Brownell argue for the role of intuition in the production of new knowledge (Bruner, 1960; King and Brownell, 1966). In particular, intuition, imagination, and inductive reasoning tend to generate new hypotheses and provide the initial framework for evaluating the veracity of these claims. Often, it is only after this initial insight that the mathematician will go back and try to carefully justify his/her conclusion. Thus, the balance between intuitive insight and more methodical analytic reasoning is an essential component of building mathematical knowledge (Bruner,1960).

Modes of Inquiry

In addition to having intuition about the truth/falsity of mathematical ideas, a knower of mathematics must have a toolbox of possible approaches and techniques to solve new problems. Thus, another characteristic of knowing mathematics is the acquisition of the common problem-solving techniques that give one entry points to tackling new problems. As King and Brownell and Schwab note, every discipline has certain "modes of inquiry" or habits of mind that reflect the common ways in which new problems are addressed in the field (King and Brownell, 1966; Schwab, 1961). These modes of inquiry may include commonly applied techniques (e.g. searching for patterns in smaller examples) or big ideas (e.g. useful theorems or representations) to apply, but they may also reflect more broadly the types of investigations pursued in that discipline. For example, in other sciences, the structure of acquiring new knowledge is the formulation of hypotheses from examination of data, new experimentation to gather further data to test the hypotheses, followed by an attempt to refine the hypotheses in light of the new data. This cycle may result in the invalidation of previous theories so that the state of understanding in science is constantly evolving and never certain (Schwab, 1961). In mathematics, however, there is no expectation that new data will cause one to discount earlier theorems since these theorems have been *proven* to be true. Furthermore, in mathematics, it is not acceptable to justify a claim based solely on example data. Thus, there is a drastically different sense in which mathematicians versus scientists "know" something.

Form of Teaching and Learning Mathematics

Given this different sense of "knowing" in different disciplines, it seems clear that the form of teaching and learning should be specialized to reflect the particulars of that discipline. In the context of mathematics, to see the potential form of this specialization, it is easiest to first describe what forms teaching and learning should *not* take. Mathematics should not be represented as a list of facts and formulae handed down from above to be memorized. It should not be construed as a collection of disjoint procedures that have no coherent origins or explanations. Finally, while we noted above that mathematical theorems are known to be true for all time, in contrast with some scientific theories which may become obsolete, they are not true simply because some supernatural force decreed them as such, but rather because individuals have constructed them from simple mathematical building blocks such as axioms and definitions. Thus, we must convey to students that mathematics, by its very nature, should always make sense, and that its basic premise is that every result should be absolutely justifiable.

Sense-making In the Classroom

Given these expectations, what form should mathematics teaching and learning actually take? First, it must emphasize sense-making and justification. This component was a cornerstone of Lampert's classroom model. She emphasized that students must be able to explain their reasoning and justify their answers to each other. It became an expectation of the students that the answers they arrived at should make sense and that it should be possible for them to determine when an answer did not meet this criterion. She did not present learning procedures as the appropriate form of "knowing" mathematics, but rather the ability to reason through the logical implications of a student's ideas or methods (Lampert, 2002).

In order to implement such a model in the classroom, Lampert argued that it is necessary to develop a classroom culture that expects and values these modes of conduct. In particular, it is essential to give students space to work through their ideas and share them with others in a non-threatening environment. Additionally, the students must become the judge of whether or not an answer is right if they are to fully appreciate the notion that mathematics is a coherent subject that any student may engage in (Lampert, 2002).

Another important component of sense-making and knowing mathematics is understanding the connections between material, specifically the connections across topics and across the various arenas in which a child may have encountered such topics. Building such connections is vital not only to cultivating sense-making in the students, but also to generating in the students a greater buy-in, i.e. a greater appreciation of the relevance or interest of the material. As Dewey explains,

> From the side of the child, it is a question of seeing how his experience already contains within itself elements – facts and truths – of just the same sort as those entering into the formulated study. (Dewey, 1902, p.189)

Tying new material into old concepts with which the child is already familiar is a valuable technique for developing the mindset that mathematics is relevant and makes sense.

Another important aspect of this is choosing appropriate representations of material so that it is accessible to the child. Wilson, Shulman and Richert give multiple examples of how the representation of material can affect the child's ability to relate to it and grasp the bigger ideas. One non-mathematical example of this was a teacher choosing to recast the conflicts that arise in the play *Julius Caesar* in terms of conflicts involving Captain Kirk in *Star Trek* (Wilson et. al., 1987). This transformation of the material captured the overarching themes that the children should grasp while framing it in a situation that was already familiar to them. Such a transformation is a hallmark of what Bruner describes as teaching a topic to a child of any age in a way that stays true to the nature of the discipline (Bruner, 1960), and is an important component of the pedagogical changes necessary to teach the discipline effectively.

Intuition In the Classroom

As outlined previously, intuition is a valuable tool for investigating mathematical ideas and generating further knowledge. In the classroom, such intuition might take the form of students making reasonable guesses at an answer or at a method for attacking a problem. Intuition together with analytic thinking might then further guide the students as they try to work out and verify/falsify these guesses to check their reasonableness.

For these skills to even appear in a classroom, though, adjustments must be made to the current mode of teaching mathematics. Students must be exposed to problems that require them to make conjectures about methods of attack; thus, we cannot simply provide them with problems that have one possible solution method, which is then made obvious by the place in the curriculum where the problem arises. Lampert advocates using problems that have multiple points of entry, so that every student may find a method for engaging with the problem (Lampert, 2002).

To further cultivate the role of intuition in developing a strategy for attacking a problem, Bruner suggests that the teacher should model the technique of making guesses then reasoning through whether or not these guesses make sense (Bruner, 1960). In other words, the teachers must introduce some uncertainty into their teaching style; they must embrace situations where students raise questions that the teachers have not thought about in advance, and use those situations to actively demonstrate their thought process for attacking novel problems.

Modes of Inquiry In the Classroom

In addition to modifying the types of problems that are presented to children, we must also modify the expectations for how they solve such problems in order to accurately capture the nature of the discipline. Lampert made such an adjustment to children's conception of doing mathematics in her classroom by encouraging students to make initial conjectures about a problem and perform their own computations to support these conjectures. She then expected them to provide reasoning to support their conjectures, discuss their conjectures with classmates, and reevaluate these conjectures in light of classmate feedback (Lampert, 2002). This period of discovery/exploration followed by dialogue characterizes the process followed by almost all practitioners of the discipline of mathematics, and thus is essential to expose to children so that they may better comprehend the modes of inquiry of the discipline (King and Brownell, 1966).

Another important aspect of understanding mathematical modes of inquiry is understanding the types of questions that a mathematician asks about a problem to push his/her knowledge further. By compiling a mental list of the types of questions asked by a mathematician, students will be better prepared to explore a new mathematical topic on their own in the future. This is precisely the purpose of education in Hawkins' view: to produce a person who can serve as his/her own teacher in the future (Hawkins, 1967).

One prerequisite to getting children to ask deep, probing questions of a new topic is that the students actually feel interested in and engaged by the material they are to investigate. This is the role of the "It" in Hawkins' model; he suggests that the role of the teacher cannot be to merely "love" the child, but rather to generate respect in the child by allowing the child to encounter/discover new material that will actually engage him/her. The teacher must then pay close attention to the responses of the child to determine what really interests him/her and build off of that (Hawkins, 1967). This same model was advocated by King and Brownell, Bruner, and Dewey as they described the classroom as a place for presenting a "planned series of encounters" for the child to become engaged with the material, and by Schwab, who saw the role of the teacher as cultivating Eros in the student (King and Brownell, 1966; Bruner, 1960; Dewey 1902; Schwab, 1954).

Conclusion

It is clear that some significant changes to the current pedagogical style of K-12 education are necessary to allow the classroom to accurately reflect the nature of the discipline of mathematics. We have outlined some necessary conditions to allow mathematical knowing to faithfully appear in the classroom, such as finding problems or topics that actively engage students and finding ways to represent the material that access/build off previous student knowledge and understanding. We have also seen aspects of Lampert's style of teaching with problems that enable her students to engage in the study of mathematics in a meaningful way. These are visions for *what* must change in the teaching and learning of mathematics, but they do not give a sufficient description of *how* these changes will occur. Cohen argues that these changes are much more difficult to implement than many educational theorists have suggested because we must battle against societal expectations for the form of teaching that have persisted for centuries. Thus, we must expect that change will progress much more slowly than previous theorists have predicted, and that implementing these changes will require a much greater scaffold of support than has previously been provided (Cohen, 1988). While it is somewhat disheartening to hear that these changes may progress on such a slow timeframe, it is valuable to be aware of these additional factors so that we may better facilitate teaching and learning change in the future.

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