

Operator Theory:

What I usually do is discuss the idea of transformation via the idea of electronic communication: think of changing a signal such as your voice into a digital format. The sound waves are sampled and then converted into sequences of 0's and 1's. This is an example of a linear transformation (where the domain is the set of all sound waves and the range is the space of all sequences of 0's and 1's). The second part is to "nearly" reconstruct the original signal, which means to take the sequence of 0's and 1's and make a sound wave which your ear cannot distinguish from the original sound of your voice. This is also (nearly) a linear transformation. While necessarily there is information loss, the hope is that you can do well enough for practical purposes.

I then tell people that I don't study these transformations themselves, but I study collections of things like them, which have certain algebraic properties. People always like real world applications (they never ask about applications of fine arts, but somehow math requires applications, rather than simply appreciating the beauty of the ideas). So as applications, I tell them that the subject arose in trying to understand aspects of quantum mechanics, but then grew to have applications in mathematical biology, engineering, and other areas of math.

-David Pitts

For someone with a bit of scientific/technical knowledge I usually say something like:

One of the big ideas of 20th c science was quantum mechanics, and the key thing quantum mechanics found was that there's a fundamental limit to how well you can measure some things. And that's different from just how accurate your instruments are - even if you could make your instruments more and more accurate you can never measure *both* something's position and speed very accurately. Even if you could measure *one* of those very very accurately, you will always find that when you try to measure both together there's an inaccuracy built right into the setup which makes one or other of the measurement inaccurate. So there's a fundamental limit on how much we can know about certain pairs of linked quantities - such as position and speed.

Physicists in the first half of the 20th c tried to find a mathematical model which showed how this could make sense and how the quantum view of the world is different from the one we had before (the "classical" view). They found it comes down to the fact that in ordinary arithmetic when you multiply two numbers, it doesn't matter what order you do it in: two times three and three times two are both the same. Skipping over a lot of math that I don't necessarily understand so well myself, at the heart of it, those two numbers commuting - a times b being the same as b times a - are like the classical view of position and speed, where two quantities are completely independent and you can measure each of them as accurately as you want.

The quantum view builds a model of algebra where a times b and b times a need not be the same and so their difference - a times b subtracted from b times a - is some non-zero quantity. That non-zero quantity represents at some level that absolute lower bound for how well we can measure two related quantities together.

Now the operator theory I study doesn't work with quantum theory directly, but does use the mathematical objects which give you this property "non-commutativity" and it looks at the algebra you get with families of these non-commuting variables together. So when people in operator theory talk about non-commutative topology, or non-commutative (or "quantum") groups, as a way of developing other parts of mathematics with the quantum mechanical perspective.

<Awkward pause>

So anyways, tell me more about tax law...?"

-John Orr

Differential Equations:

Differential equations arise in many different areas of the physical sciences and engineering and also in diverse subjects such as medicine, psychology, economics, operations research to mention just a few. As simple examples, differential equations arise in the theory of electric circuits, gravitational equilibrium of a star, learning of tasks, vibrating strings, heat flow, wave motion, and so on.

-Lynn Erbe

In the sciences and engineering, mathematical models are developed to aid in the understanding of physical phenomena. These models often yield an equation that contains some derivatives of an unknown function. Such an equation is called a differential equation. Two examples of models developed in calculus are the free fall of a body and the decay of a radioactive substance.

-“Fundamentals of Differential Equations” by Nagle, Saff, and Snider

Coding Theory:

I typically say something like, "There are many examples of instances in which information needs to be transmitted reliably across a channel --- satellite pictures from outer space, cellular (or digital) phone conversations, email, etc.; even data storage (on a hard disk or a flash drive or compact disc, for example) can be thought of in this way. No matter how careful we are, errors are bound to occur. It is the goal of coding theory to find ways of adding redundancy to the information so that these errors can be detected and efficiently corrected."

-Judy Walker

Math Education:

Mathematics education research has often been associated with innovations in curriculum such as calculus reform, statistics education, and the use of computational and graphing technology in instruction. This association has had several unfortunate consequences (linking all math ed research with reform agendas and "teaching experiments"). Most simply, mathematics education research is inquiry by carefully developed research methods aimed at providing evidence about the nature and relationships of many mathematics learning and teaching phenomena. It seeks to clarify the phenomena, illuminate them, explain how they are related to other phenomena, and how this may be related to undergraduate mathematics course organization and teaching. -"Mathematics Education Research: A Guide for the Research Mathematician" by McNight², Magid, & Murphy

Functional Integration:

You're doing Calculus but instead of using points on the real line (or complex plane), your points are functions. So you're integrating functions of functions (called functionals). How to explain Calculus to someone who's never had it? Calculus makes hard problems easy. It is useful for finding max/mins in real applications.

-Dave Skoug (transcribed)

Commutative Algebra and Algebraic Geometry:

If I want to explain very briefly what I do, I say that I work with different number systems and finding solutions to systems of equations in these number systems. They always like when you mention applications, so I lie a little and say that what I do has applications to coding theory (cell phone transmissions and secure internet transactions). If I have more time or a chalkboard, I like to describe the geometric connections: you can think of a line in 3-space as the intersection of two planes. What about a surface in 4-space? Is it the intersection of two 3-dimensional objects?

-Tom Marley (transcribed)

The motivating goal in Algebraic Geometry is to understand solutions sets of systems of polynomial equations in several variables. Algebraic Geometry thus subsumes linear algebra, where the goal is to understand solutions sets of systems of linear equations in several variables, but non-linear equations turn out to be much harder to deal with.

Finding solutions of polynomial equations has been of interest for thousands of years. The Babylonians could solve quadratic equations, but it wasn't until 3200 years later that formulas were found, by the Italians, for solving cubics and quartics. By the 1800s it was finally understood no such formulas existed for higher degree equations in a single variable, and hence that there was little hope for explicit solutions for systems of polynomial equations in several variables. The focus in Algebraic Geometry is thus mostly on finding qualitative information about a solution set: is it non-empty, if so what is its dimension, how many different pieces does it decompose into, what shape does each piece have, etc. From this basic root have grown many branches, so today Algebraic Geometry is very diverse, including tendrils into commutative algebra, complex analysis, topology, category theory and even high energy physics!

-Brian Harbourne

Math Biology and Neuroscience:

Whenever I say Math Bio, the response is usually a surprise that biology uses mathematics! So, I try to talk about the large amounts of data collection, using Ecology as an example (most people have learned about ecology and are more comfortable with it than DNA sequencing, for example), and get them to realize these are things they already knew Biologists did, and so using Mathematics should be obvious. Then I explain that as a math biologist, I try to put together all of this vast amount of data and use it to create equations that will be able to make predictions about the future of the ecosystem. They usually think then that I'm a biology statistician, but it's hard to get across the idea of a model (as opposed to analyzing data from things that have already happened).

There are a few examples I use to help with the concept of modeling: pest outbreaks (predicting the next grasshopper boom), global climate change (what will happen to animals if the temperature increases by 3 degrees), managing a fishery (what is the optimal number of fish to catch each year). I also draw on the fact that they may remember some elementary physics, and explain that $F=m*a$ is an equation that models real life, and it is equations like this that I am working to develop (with respect to biology). -David Logan (transcribed)

A central challenge in neuroscience is to understand how neurons represent and process information about the world. Our brain is constantly constructing and updating a representation of the world around us, largely from sensory inputs (visual, auditory, tactile, etc.). How is this information represented in the brain?

This requires understanding how neurons - and, more generally, populations of neurons working together in neural networks - encode and transform information that comes in through the senses.

Mathematics is used in (at least) two ways to address this challenge.

(1) Mathematical methods are used for analyzing data obtained from experiments, where the activity of populations of neurons is recorded. How is the observed neural activity related to sensory stimuli? Finding "rules" discovering these relationships in noisy data requires a lot of mathematical analysis.

(2) Mathematical models are used to study the properties of single neurons and also networks of neurons. These models are usually highly simplified, but capture important properties of the biology in a (hopefully) mathematically tractable way. We know very little about how networks of neurons behave, but mathematical models of these networks give us a valuable tool for exploring network dynamics and relating this to experimental observations. Occasionally, a useful prediction can also be obtained from this kind of theoretical study. (A prediction is "useful" if it motivates an interesting experiment where we learn something. Sometimes incorrect predictions are useful!)

In my own research, I do both (1) and (2), and the two "types" of work often inform each other.

-Carina Curto

Graph Theory and Combinatorics:

I actually don't have much success in explaining my area to non-mathematicians (or nontechnical) people. Usually I say that I study the structure of networks, using the specific example of the Internet and computer networks.

-Stephen Hartke

Combinatorics is a fancy word for counting. Combinatorics is concerned with determining the number of logical possibilities of some event without necessarily listing all the particular outcomes. One can often perform calculations involving probabilities simply by counting the possible outcomes. Combinatorics often requires counting the number of rearrangements or groupings of a set of objects.

-MathCounts

Groups, Semigroups, and Topology:

A fair bit of my work is concerned with the general area of algorithmic problems in algebra. Most people who have seen any mathematics at all have come across the concept of an "algorithm" (i.e. some kind of finite step by step process or list of instructions) for answering many questions in mathematics. For example, in ancient times Euclid provided an algorithm for finding the greatest common divisor of two integers, and at some level algorithms permeate virtually all fields of modern mathematics and its applications. Of course, modern computing machines make essential use of often very sophisticated mathematical algorithms to do anything that they are able to do. Indeed, up until the middle of the 20th century, many mathematicians believed that there must be constructive, "algorithmic" solutions to all mathematical problems. In fact at the International Congress of Mathematicians in Paris in 1900 the great German mathematician David Hilbert posed a collection of 23 famous problems that drove much of 20th century mathematics. Several of Hilbert's problems essentially called for finding algorithms to provide solutions to mathematical questions. For example, Hilbert's 10th problem asks for an algorithm to determine whether an arbitrary diophantine equation (a polynomial equation with integer coefficients) does or does not have a solution in integers. During the first half of the 20th century, mathematicians and logicians began to ask whether there might actually be problems for which it could be proved that no algorithmic solution could exist. The question could not be answered without an agreed upon definition of exactly what an "algorithm" is, and this led to fundamental work by mathematicians such as Turing, Markov, Post, Church etc, who gave rigorous definitions of the concept of an algorithm. Then people began to prove that

many famous outstanding problems in mathematics in fact admit no algorithmic solution. For example, in 1970, Yuri Matiyasevich, building on earlier work of Julia Robinson and Hilary Putnam and others, was able to show that Hilbert's 10th problem is in fact undecidable (i.e. there is (provably) no algorithm for making the decision as to whether arbitrary diophantine equations admit integer solutions).

Algorithmic problems are common throughout many areas of modern mathematics. One such problem that is very well studied in group theory and semigroup theory is the "word problem". Given a finite set of "generators" for the semigroup (i.e. elements such that all elements of the semigroup may be expressed as products of the generators) and a finite set of "relations" (i.e. various constraints that the generators must satisfy), is there an algorithm to decide, on input an arbitrary pair of elements of the semigroup expressed in terms of the generators, whether or not they are equal in the semigroup - i.e. whether one element may be transformed to the other via some sequence of applications of the relations? The word problem for arbitrary semigroups was shown to be undecidable by Post in the 1950's and the word problem for arbitrary groups was shown to be undecidable independently by Boone and Novikov in the 1960's, but these problems remain open for many special classes of groups and semigroups. Even when an algorithmic problem such as the word problem (or many other such problems) is shown to be decidable, there is a serious question as to how "practical" the algorithmic solution is, and this leads to the theory of computational complexity of algorithms and algorithmic problems. This kind of question is at the heart of much work in many areas of mathematics and theoretical computer science.

-John Meakin

When you have an object and you bend it a little bit (or a lot!), the object hasn't really inherently changed; topology is about understanding what it is about the object that hasn't changed. As an example, what makes a ball a ball, and not a doughnut? Knot theory is really the topology of knotted circles; what is different about your shoes when they are tied as opposed to when they aren't? How many different ways can you tie your shoes? Since lots of physical objects can be modeled as knotted objects (molecules, elementary particles="strings"), the concepts and techniques of knot theory can be applied to many physical systems.

-Mark Brittenham

2÷		90×			7+
2-	2÷		11+		
	1-			7+	2-
5-	3-				
	5+			30×	
3÷		8+			