1. (36 points) Evaluate each of the following integrals. (You must show all of your work to receive full credit. Here, no calculators are allowed.)

\[ \int \frac{x}{\sqrt{x+1}} \, dx \quad (a) \quad \int \frac{\pi}{2} \sin^4 x \cos x \, dx \quad (b) \quad \int \frac{e^{2x}}{1 + e^{4x}} \, dx \quad \text{(c)} \quad \int \frac{1}{x \ln x} \, dx \quad \text{(d)} \]

2. (16 points, 8 points each) A cylindrical tank of radius 4 feet and height 3 feet is full of fluid whose density at height \( x \) feet from the bottom of the tank is given by \( \rho(x) = \frac{2}{\sqrt{x+1}} \) lb/ft\(^3\).

(a) Find a Riemann sum whose value approximates the weight of the fluid.
(b) Write down and evaluate an integral whose value is exactly the weight of the fluid.

3. (20 points) A construction worker pulls a 100-pound motor from ground level to the top of a 50-foot-high building using a rope that weighs \( \frac{1}{2} \) lb/ft. Let \( W_{\text{rope}} \) and \( W_{\text{motor}} \) denote respectively the work done in lifting the rope (alone) and the motor (alone) to the top of the building.

(a) Find a Riemann sum whose value approximates \( W_{\text{rope}} \).
(b) Write down and evaluate an integral whose value is exactly \( W_{\text{rope}} \).
(c) What is the value of \( W_{\text{motor}} \)?

4. (24 points, 8 points each) A fish tank has a square base whose length is 6 feet and rectangular sides of height 2 feet. Assume that the tank is filled with water weighing \( \rho = 62.4 \) lb/ft\(^3\).

(a) Find a Riemann sum whose value approximates the work required to pump all of the water over the top of the tank.
(b) Write down but do not evaluate an integral whose value is exactly the work required to pump all of the water over the top of the tank.
(c) Write down but do not evaluate an integral whose value is exactly the force exerted by the water on one side of the tank.

5. (12 points) Find and evaluate an integral whose value gives the volume of the solid obtained by revolving the bounded region between the curves \( y = \sqrt{x} \) and \( y = x^2 \) in the first quadrant about the \( x \)-axis.

6. Write down but do not evaluate an integral giving the volume of the torus obtained by rotating the curve \( (x - 5)^2 + y^2 = 9 \) about the \( y \)-axis.

7. A rod of length 2 meters is lying on the \( x \)-axis on the interval \( [0, 2] \). Assume its density \( x \) meters from the origin is given by \( \rho(x) = 5 + 4 \cos(2x) \) kg/m.

(a) (8 pts) Find a Riemann sum whose value approximates the mass of the rod.
(b) (8 pts) Write down (don’t evaluate) an integral whose value is exactly the mass of the rod.
(c) Write down but do not evaluate an integral whose value is the first moment of the rod.

8. Chapter 4 review (p. 398): 11, 13, 15, 17, 19, 41, 46, 53, 54, 55, 56, 57

9. Chapter 5 review (p. 475): 1, 7, 11, 15, 17, 33, 35, 37, 38

10. Chapter 6 review (p. 552): 25, 41, 43, 45a, 57, 61
11. (a) Make the substitution \( u = x^4 + 2 \) in the integral \( \int_0^2 x^3 f(x^4 + 2) \, dx \).

(b) If \( \int_0^1 f(t) \, dt = 2 \), \( \int_0^2 f(t) \, dt = 1 \), \( \int_0^{10} f(t) \, dt = 7 \), \( \int_0^{16} f(t) \, dt = -5 \), and \( \int_0^{18} f(t) \, dt = -11 \), what is the value of \( \int_0^2 x^3 f(x^4 + 2) \, dx \)?

12. An unusual fishtank is made with a width of 6 feet and a cross-section (perpendicular to the 6-foot dimension) given by the region bounded by the \( x \)-axis and the curve \( y = 1 - \frac{1}{4}x^2 \).

(a) Sketch a picture of the the tank.

(b) Find and evaluate an integral giving the cross-sectional area.

(c) What is the volume of the tank?

(d) Find and evaluate an integral giving the amount of work required to pump all the water out of a full tank (through the top of the tank).

(e) Find and evaluate an integral giving the hydrostatic force exerted by the water on one of the flat ends of the tank.

13. Evaluate the following integrals:

(a) \( \int (3x + 1)^2 \sqrt{4x + 1} \, dx \)  

(b) \( \int \left( x + \frac{2}{x} \right)^2 \, dx \)
Answers

1. (a) \[ \frac{2}{3} (x + 1)^{3/2} - 2(x + 1)^{1/2} + C \] (b) \[ \frac{1}{5} \] (c) \[ \frac{1}{2} \tan^{-1} e^{2x} + C \] (d) \[ \ln |\ln x| + C \]

2. (a) \[ \sum_{k=1}^{n} \frac{32\pi}{\sqrt{z_k + 1}} \Delta x, \text{ where } x_{k-1} \leq z_k \leq x_k, \] (b) \[ \int_{0}^{3} \frac{32\pi}{\sqrt{x + 1}} \, dx = 64\pi \text{ lbs} \approx 201 \text{ lbs} \]

3. (a) \[ W_{\text{rope}} \approx \sum_{k=1}^{n} \frac{1}{2}(50 - z_k)\Delta y \] or \[ W_{\text{rope}} \approx \sum_{k=1}^{n} \frac{1}{2}z_k\Delta y, \text{ where } y_{k-1} \leq z_k \leq y_k \] (b) \[ W_{\text{rope}} = \int_{0}^{50} \frac{1}{2}(50 - y) \, dy = 625 \text{ ft-lbs} \] or \[ W_{\text{rope}} = \int_{0}^{50} \frac{1}{2}y \, dy = 625 \text{ ft-lbs} \]

4. (a) \[ \sum_{k=1}^{n} (62.4)(36) z_k \Delta y \] or \[ \sum_{k=1}^{n} (62.4)(36)(2 - z_k) \Delta y, \text{ where } y_{k-1} \leq z_k \leq y_k \] (b) \[ \int_{0}^{2} (62.4)(36)y \, dy = 4492.8 \text{ ft-lbs} \] or \[ \int_{0}^{2} (62.4)(36)(2 - y) \, dy = 4492.8 \text{ ft-lbs} \]

5. By washers: \[ \int_{0}^{\pi/2} \left[ \pi(x^{1/2})^2 - \pi(x^2)^2 \right] \, dx = \pi \int_{0}^{1} (x - x^4) \, dx = \frac{3}{10} \pi \approx 0.94 \]

6. By shells: \[ \int_{-3}^{3} 2\pi y \sqrt{9 - (x - 5)^2} \, dy = 2\pi \int_{0}^{4} (y^{3/2} - y^3) \, dy = \frac{3}{10} \pi \approx 0.94 \]

7. (a) \[ \sum_{k=1}^{n} \rho(z_k) \Delta x = \sum_{k=1}^{n} [5 + 4\cos(2z_k)] \Delta x, \text{ where } x_{k-1} \leq z_k \leq x_k \] (b) \[ \int_{0}^{2} \rho(x) \, dx = \int_{0}^{2} [5 + 4\cos(2x)] \, dx \]

8. #46: \[ \frac{8}{3} \] #54: \[ \frac{1}{2}(\ln 2)^2 \] #56: 6. For the odds, see the back of the book.

9. #38: 46,800 lbs. For the odds, see the back of the book.

10. See the back of the book.

11. (a) \[ \frac{1}{4} \int_{2}^{18} f(u) \, du \]

(b) Since \[ \int_{2}^{18} f(t) \, dt = \int_{0}^{18} f(t) \, dt - \int_{0}^{2} f(t) \, dt = -11 - 1 = -12, \] we have \[ \frac{1}{4} \int_{2}^{18} f(u) \, du = \frac{1}{4}(-12) = -3. \]

12. (b) \[ \int_{2}^{1} (1 - \frac{1}{4}x^2) \, dx = \frac{8}{3} \text{ ft}^2 \] or \[ \int_{0}^{1} 4\sqrt{1 - y} \, dy = \frac{8}{3} \text{ ft}^2 \]

(c) \[ \left( \frac{3}{2} \text{ ft}^2 \right) (6 \text{ ft}) = 16 \text{ ft}^3 \]

(d) \[ \int_{0}^{1} (62.4)24\sqrt{1 - y} \, dy = 1497.6 \int_{0}^{1} (1 - y)^{3/2} \, dy = 599.04 \text{ ft-lbs} \]

(e) \[ \int_{0}^{1} (62.4)4\sqrt{1 - y} \, dy = 249.6 \int_{0}^{1} (1 - y)^{3/2} \, dy = 99.84 \text{ lbs} \]

13. (a) \[ \frac{9}{100}(4x + 1)^{13/6} + \frac{3}{50}(4x + 1)^{7/6} + C \] (b) \[ \frac{1}{2}x^3 + 4x - \frac{4}{x} + C \]