1. Let \( f(x) = x^5 + 2x^3 + 5x - 10 \).
   
   (a) How can you be sure \( f(x) \) has an inverse? (It does.)
   (b) What is \( f^{-1}(-2) \)?
   (c) What is \( \left[ \frac{d}{dx} f^{-1}(x) \right]_{x=-2} \) (the derivative of \( f^{-1}(x) \) at \( x = -2 \))? 

2. Suppose Dragline gives his buddy Luke a cold drink at 45°F. After 5 minutes in the hot 93°F sun, the drink has warmed to 65°F. When will the temperature of the drink reach 70°F?

3. Let \( f(t) \) be the number of centimeters of rainfall that has fallen since midnight, where \( t \) is the time in hours. Interpret the following in practical terms, giving units.
   
   (a) \( f(10) = 3.1 \)
   (b) \( f^{-1}(10) = 16 \)
   (c) \( f'(8) = 0.4 \)
   (d) \( (f^{-1})'(5) = 2 \)

4. Find the inverses of the following functions:
   
   \( f(x) = \frac{2x + 6}{3x - 5} \), \( g(x) = (2x - 3)^8 \), \( j(x) = \frac{7}{4} \ln(2x + 7) - 9 \).

5. Let \( P = f(t) \) be the population of Stillwater, Minnesota (in thousands) \( t \) years after 1980. Suppose that \( f \) is a linear function and that the population was 18,000 in 1985 and 21,000 in 1989.
   
   (a) Find a formula for \( f(t) \).
   (b) Find a formula for \( f^{-1}(P) \). Give an interpretation of the function \( f^{-1} \).
   (c) Does it make sense to swap the variables and write \( P = f^{-1}(t) \)?

6. If \( \theta \) is an angle in the fourth quadrant and \( \cos \theta = \frac{2}{3} \), what is \( \sin \theta \)? \( \tan \theta \)? \( \sec \theta \)? \( \csc \theta \)? \( \cot \theta \)? (Find exact answers.)

7. Evaluate the following exactly, without using a calculator:
   
   (a) \( \sin(\arcsin(.123)) \), (b) \( \tan(\arcsin(\frac{4}{5})) \), (c) \( \arcsin(\pi) \).

8. Let \( f(x) = \frac{x}{x+1} \). Evaluate and simplify:
   
   (a) \( f \left( \frac{1}{2} \right) \), (b) \( \frac{1}{f(x)} \), (c) \( f^{-1}(x) \).

9. A population of bacteria which is growing exponentially has a doubling time of 1 minute. After 30 minutes, there are 50 million bacteria. When will there be 100 million bacteria? By what percentage does the population increase in one second? Find a formula for \( P(t) \), the size of the population after \( t \) seconds.

10. A picture supposedly painted by Vermeer (1632-1675) contains 99.5% of its original carbon-14. Could the picture be genuine?

11. How much work is required to pull a one kilogram object up 50 meters by a cord which has a mass of 9 grams per meter? Assume you’re on the moon, standing on a ledge 120 meters above the object.

   Note that this problem uses metric units, but your book uses English units in every problem. To use metric units, recall that a kilogram is a unit of mass (such as the English unit “slugs”). To convert this to a unit of force (weight), multiply by the acceleration of gravity (1.6 m/s² on the moon): \( (1 \text{ kg}) \times (1.6 \text{ m/s}^2) = 1.6 \text{ Newton; thus a “Newton” is the metric equivalent of the English “pound (force).”} \)
12. The soot produced by a garbage incinerator spreads out in a circular pattern. The depth, $H(r)$, in millimeters, of the soot deposited each month at a distance $r$ kilometers from the incinerator is given by $H(r) = 0.115e^{-2r}$. Write an integral giving the total volume of soot deposited within 5 kilometers of the incinerator each month. How much is deposited in cubic meters?

13. Springfield has a population of 10,000 people and is growing at a rate of 11.2% per year. Shelbyville has a population of 50,000 people and is declining by 1,959 people per year.

   (a) Find formulas for the populations of Springfield $S(t)$ and Shelbyville $H(t)$ after $t$ years. When will the two populations be equal? Explain how you got your answer.

   (b) What is the doubling time of Springfield?