

Name: \_\_\_\_\_ Day/Time Class Meets: \_\_\_\_\_

**Instructions:**

- Use pencil only.
- Work problems completely and clearly in the space provided. Place your answer(s) on the line(s) provided.
- Use backs of sheets for additional writing space, if necessary.
- Credit (partial or full) will be given only if sufficient steps leading to the solution are shown. In most problems, no credit will be given for answers only.
- No books or notes may be used on the exam.
- This exam has a total of 5 pages (including the cover page). Check to make sure you have a complete exam.
- Erase or cross out work that you do not want graded.

1. Write the following set in interval notation, and then graph the interval (write the interval on the line and draw the graph in the free space):

$$\{x \mid -2 < x \leq 4\}.$$

**Solution:**

$$(-2, 4]$$

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2. Solve the following:

(a)  $3(x + 6) = 5x - 8$

**Solution:**

$$\begin{aligned} 3(x + 6) &= 5x - 8 \\ \Rightarrow 3x + 18 &= 5x - 8 \\ \Rightarrow 26 &= 2x \\ \Rightarrow x &= 13 \end{aligned}$$

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(b)  $-4 = 2x(6x + 7)$

**Solution:**

$$\begin{aligned} -4 &= 2x(6x + 7) \\ \Rightarrow -4 &= 12x^2 + 14x \\ \Rightarrow 0 &= 12x^2 + 14x + 4 \\ \Rightarrow 0 &= 12x^2 + 6x + 8x + 4 \\ \Rightarrow 0 &= 6x(2x + 1) + 4(2x + 1) \\ \Rightarrow 0 &= (6x + 4)(2x + 1) \\ \Rightarrow x &= -2/3 \text{ and } x = -1/2 \end{aligned}$$

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3. Find the midpoint of the segment with endpoints  $(4, -16)$  and  $(8, 2)$ .

**Solution:**

$$M = \left( \frac{8+4}{2}, \frac{-16+2}{2} \right) = \left( \frac{12}{2}, \frac{-14}{2} \right) = (6, -7)$$

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4. Simplify

$$\frac{36x^4y^2}{9x^{-2}y}$$

**Solution:**

$$\frac{36x^4y^2}{9x^{-2}y} = 4x^4x^2y^2y^{-1} = 4x^{4+2}y^{2-1} = 4x^6y$$

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5. Convert  $\sqrt[3]{27xy^4z^6}$  to exponential form and simplify.

**Solution:**

$$\begin{aligned}\sqrt[3]{27xy^4z^6} &= (27xy^4z^6)^{1/3} \\ &= 27^{1/3}x^{1/3}y^{4/3}z^{6/3} \\ &= 3x^{1/3}y^{4/3}z^2\end{aligned}$$

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6. Perform the following operations, and if necessary, simplify:

(a)  $(x+2)(x^2+3x-9)$

**Solution:**

$$\begin{aligned}(x+2)(x^2+3x-9) &= x^2(x+2) + 3x(x+2) - 9(x+2) \\ &= x^3 + 2x^2 + 3x^2 + 6x - 9x - 18 \\ &= x^3 + 5x^2 - 3x - 18\end{aligned}$$

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(b)  $\frac{x^2+5x+6}{2x-4} \cdot \frac{8x-16}{x^2-x-12}$

**Solution:**

$$\begin{aligned}\frac{x^2+5x+6}{2x-4} \cdot \frac{8x-16}{x^2-x-12} &= \frac{(x+3)(x+2)}{2(x-2)} \cdot \frac{8(x-2)}{(x-4)(x+3)} \\ &= \frac{x+2}{4(x-4)}\end{aligned}$$

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(c)  $\frac{3}{y+3} - \frac{y+2}{y^2-9}$

**Solution:**

$$\frac{3}{y+3} - \frac{y+2}{y^2-9} = \frac{3(y-3)}{(y-3)(y+3)} - \frac{y+2}{(y-3)(y+3)} = \frac{3y-9-y-2}{(y-3)(y+3)} = \frac{2y-11}{(y-3)(y+3)}$$

7. Factor  $z^3 + 6z^2 - 3z - 18$ .

**Solution:**

$$z^3 + 6z^2 - 3z - 18 = z^2(z + 6) - 3(z + 6) = (z^2 - 3)(z + 6)$$

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8. Rationalize the denominator of the rational expression

$$\frac{3}{\sqrt{x} - 2}$$

**Solution:**

$$\frac{3}{\sqrt{x} - 2} = \frac{3}{\sqrt{x} - 2} \cdot \frac{\sqrt{x} + 2}{\sqrt{x} + 2} = \frac{3(\sqrt{x} + 2)}{x - 4}$$

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9. Find the distance between the points  $(-5, 6)$  and  $(-9, 3)$ . Give an exact answer.

**Solution:**

$$\begin{aligned} d &= \sqrt{(-9 - (-5))^2 + (3 - 6)^2} \\ &= \sqrt{(-4)^2 + (-3)^2} \\ &= \sqrt{16 + 9} \\ &= \sqrt{25} = 5 \end{aligned}$$

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10. Find the domain of the following:

(a)  $f(x) = \frac{1}{x^2 + 5x + 4}$

**Solution:**

$$\begin{aligned} &x^2 + 5x + 4 = 0 \\ \Rightarrow &(x + 4)(x + 1) = 0 \\ \Rightarrow &x + 4 = 0 \text{ and } x + 1 = 0 \\ \Rightarrow &x = -4 \text{ and } x = -1 \\ \Rightarrow &\text{Domain of } f = (-\infty, -4) \cup (-4, -1) \cup (-1, \infty) \end{aligned}$$

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(b)  $f(x) = \sqrt{8x - 2}$ .

**Solution:**

$$\begin{aligned} &8x - 2 \geq 0 \\ \Rightarrow &8x \geq 2 \\ \Rightarrow &x \geq 1/4 \\ \Rightarrow &\text{Domain of } f = [1/4, \infty) \end{aligned}$$

11. Find the equation of a circle centered at  $(3, 2)$  and passing through the point  $(7, 5)$ .

**Solution:**

$$\begin{aligned} r &= \sqrt{(7-3)^2 + (5-2)^2} \\ &= \sqrt{4^2 + 3^2} \\ &= \sqrt{16+9} \\ &= \sqrt{25} = 5 \\ \Rightarrow \text{Equation of the circle } &(x-3)^2 + (y-2)^2 + 25 \end{aligned}$$

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12. Given that  $g(x) = 5x^2 + 3x$ , find the following and simplify:

(a)  $g(3)$

**Solution:**

$$\begin{aligned} g(3) &= 5(3)^2 + 3(3) \\ &= 5(9) + 9 \\ &= 45 + 9 \\ &= 54 \end{aligned}$$

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(b)  $g(y-1)$

**Solution:**

$$\begin{aligned} g(y-1) &= 5(y-1)^2 + 3(y-1) \\ &= 5(y^2 - 2y + 1) + 3y - 3 \\ &= 5y^2 - 10y + 5 + 3y - 3 \\ &= 5y^2 - 7y + 2 \\ &= 5y^2 - 5y - 2y + 2 \\ &= 5y(y-1) - 2(y-1) \\ &= (5y-2)(y-1) \end{aligned}$$

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