

Name: _____ Day/Time Class Meets: _____

Instructions:

- Use pencil only.
- Work problems completely and clearly in the space provided. Place your answer(s) on the line(s) provided.
- Use backs of sheets for additional writing space, if necessary.
- Credit (partial or full) will be given only if sufficient steps leading to the solution are shown. In most problems, no credit will be given for answers only.
- No books or notes may be used on the exam.
- This exam has a total of 5 pages (including the cover page). Check to make sure you have a complete exam.
- Erase or cross out work that you do not want graded.

1. Find the midpoint of the segment with endpoints $(8, -12)$ and $(6, 4)$.

Solution:

$$M = \left(\frac{8+6}{2}, \frac{-12+4}{2} \right) = \left(\frac{14}{2}, \frac{8}{2} \right) = (7, 4).$$

2. Solve the following:

(a) $4(x + 6) = 9x - 1$

Solution:

$$\begin{aligned} 4(x + 6) &= 9x - 1 \\ \Rightarrow 4x + 24 &= 9x - 1 \\ \Rightarrow 25 &= 5x \\ \Rightarrow x &= 5. \end{aligned}$$

(b) $4 = 2x(6x + 1)$

Solution:

$$\begin{aligned} 4 &= 2x(6x + 1) \\ \Rightarrow 4 &= 12x^2 + 2x \\ \Rightarrow 0 &= 12x^2 + 2x - 4 \\ \Rightarrow 0 &= 12x^2 - 6x + 8x - 4 \\ \Rightarrow 0 &= 6x(2x - 1) + 4(2x - 1) \\ \Rightarrow 0 &= (6x + 4)(2x - 1) \\ \Rightarrow 6x + 4 = 0 & \quad 2x - 1 = 0 \\ \Rightarrow x = -2/3 & \quad x = 1/2 \end{aligned}$$

3. Write the following set in interval notation, and then graph the interval (write the interval on the line and draw the graph in the free space):

$$\{x \mid -4 \leq x < 0\}.$$

Solution:

$$[-4, 0)$$

4. Perform the following operations, and if necessary, simplify:

(a) $(x + 3)(x^2 + 4x - 2)$

Solution:

$$\begin{aligned} (x + 3)(x^2 + 4x - 2) &= x^2(x + 3) + 4x(x + 3) - 2(x + 3) \\ &= x^3 + 3x^2 + 4x^2 + 12x - 2x - 6 \\ &= x^3 + 7x^2 + 10x - 6 \end{aligned}$$

(b) $\frac{x^2 + 6x + 8}{2x - 6} \cdot \frac{6x - 18}{x^2 - 3x - 10}$

Solution:

$$\begin{aligned} \frac{x^2 + 6x + 8}{2x - 6} \cdot \frac{6x - 18}{x^2 - 3x - 10} &= \frac{(x + 4)(x + 2)}{2(x - 3)} \cdot \frac{6(x - 3)}{(x - 5)(x + 2)} \\ &= \frac{3(x + 4)}{x - 5} \end{aligned}$$

(c) $\frac{3}{y + 2} - \frac{y + 1}{y^2 - 4}$

Solution:

$$\begin{aligned} \frac{3}{y + 2} - \frac{y + 1}{y^2 - 4} &= \frac{3(y - 2)}{(y + 2)(y - 2)} - \frac{y + 1}{(y - 2)(y + 2)} \\ &= \frac{3y - 6 - y - 1}{(y - 2)(y + 2)} \\ &= \frac{2y - 7}{(y - 2)(y + 2)} \end{aligned}$$

5. Simplify

$$\frac{36x^4y^2}{9x^{-2}y}$$

Solution:

$$\begin{aligned} \frac{36x^4y^2}{9x^{-2}y} &= 4x^4x^2y^2y^{-1} \\ &= 4x^{4+2}y^{2-1} \\ &= 4x^6y \end{aligned}$$

6. Convert $\sqrt[3]{27xy^4z^6}$ to exponential form and simplify.

Solution:

$$\begin{aligned}\sqrt[3]{27xy^4z^6} &= (27xy^4z^6)^{1/3} \\ &= 27^{1/3}x^{1/3}y^{4/3}z^{6/3} \\ &= 3x^{1/3}y^{4/3}z^2\end{aligned}$$

7. Find the distance between the points (3, 2) and (7, 5). Give an exact answer.

Solution:

$$\begin{aligned}d &= \sqrt{(7-3)^2 + (5-2)^2} \\ &= \sqrt{4^2 + 3^2} \\ &= \sqrt{16 + 9} \\ &= \sqrt{25} = 5\end{aligned}$$

8. Rationalize the denominator of the rational expression

$$\frac{2}{\sqrt{y} + 3}$$

Solution:

$$\begin{aligned}\frac{2}{\sqrt{y} + 3} &= \frac{2}{\sqrt{y} + 3} \cdot \frac{\sqrt{y} - 3}{\sqrt{y} - 3} \\ &= \frac{2(\sqrt{y} - 3)}{y - 9}\end{aligned}$$

9. Factor $z^3 + 6z^2 - 3z - 18$.

Solution:

$$\begin{aligned}z^3 + 6z^2 - 3z - 18 &= z^2(z + 6) - 3(z + 6) \\ &= (z^2 - 3)(z + 6)\end{aligned}$$

10. Find the domain of the following:

(a) $f(x) = \frac{1}{x^2 + 6x + 5}$

Solution:

$$\begin{aligned} & x^2 + 6x + 5 = 0 \\ \Rightarrow & (x + 1)(x + 5) = 0 \\ \Rightarrow & x = -1 \text{ and } x = -5 \\ \Rightarrow & \text{Domain of } f = (-\infty, -5) \cup (-5, 1) \cup (1, \infty) \end{aligned}$$

(b) $f(x) = \sqrt{9x - 3}$.

Solution:

$$\begin{aligned} & 9x - 3 \geq 0 \\ \Rightarrow & 9x \geq 3 \\ \Rightarrow & x \geq 1/3 \\ \Rightarrow & \text{Domain of } f = [1/3, \infty) \end{aligned}$$

11. Given that $h(x) = 4x^2 + 2x$, find the following and simplify:

(a) $g(4)$

Solution:

$$\begin{aligned} g(4) &= 4(4)^2 + 2(4) \\ &= 4(16) + 8 \\ &= 64 + 8 \\ &= 72. \end{aligned}$$

(b) $g(y - 1)$

Solution:

$$\begin{aligned} g(y - 1) &= 4(y - 1)^2 + 2(y - 1) \\ &= 4(y^2 - 2y + 1) + 2y - 2 \\ &= 4y^2 - 8y + 4 + 2y - 2 \\ &= 4y^2 - 6y + 2 \\ &= 2(2y - 1)(y - 1) \end{aligned}$$

12. Find the equation of a circle centered at $(4, 2)$ and passing through the point $(7, -2)$.

Solution:

$$\begin{aligned} r &= \sqrt{(7-4)^2 + (-2-2)^2} \\ &= \sqrt{3^2 + (-4)^2} \\ &= \sqrt{9+16} \\ &= \sqrt{25} = 5 \\ \rightarrow \text{Equation of the circle is } (x-4)^2 + (y-2)^2 &= 25 \end{aligned}$$
