An eulerian digraph is a graph that has a closed walk that visits each edge exactly once.

**Applications:** Eulerian circuits are used in routing problem (garbage collecting, mail routes, painting lines on roadways, checking parking meters, etc) as well as in bioinformatics to re-construct DNA sequences from its fragments.

**Colored Eulerian Digraphs**

**Def.** A colored eulerian digraph is an eulerian digraph with a fixed edge coloring. A compatible circuit is an eulerian circuit of G such that no two consecutive edges in the tour have the same color (i.e. no monochromatic transitions).

**The Big Question**

When does a colored eulerian digraph have a compatible circuit?

**Graphs with no compatible circuits**

**Necessary Conditions**

**Def.** Let $\gamma(v)$ be the size of the largest color class at v.

**Note:** If $\gamma(v) > \text{deg}^+(v)$ then G has no compatible circuit.

**Thm.** [Kotzig, 1968] If G is a colored eulerian undirected graph and $\gamma(v) \leq \text{deg}(v)/2$ then G has a compatible circuit.

This condition is not sufficient for digraphs.

**Splitting Procedure when $\gamma(v) = \text{deg}^+(v)$**

**Fixable Vertices**

Let $T$ be an eulerian trail of G and v a vertex. Let $S_1, S_2, \ldots, S_d$ be the segments of the trail between occurrences of v. The segments $S_1, \ldots, S_d$ are called excursions.

**Def.** A vertex v is fixable if and only if it does not have the form:

$Idea$ Iteratively fix at each vertex.

**Prop.** If every vertex is fixable then G contains a compatible circuit.

**Prop.** A vertex is fixable if and only if it does not have the form:

**Rainbow Spanning trees**

**Problem:** Let $H$ be a multigraph whose edge set is the disjoint union of 2-trails. Does there exist a subset $E'$ of the edges such that

1. $E'$ contains at most one edge from each 2-trail, and
2. the spanning subgraph with edge set $E'$ is connected?

If H has such a subset we say H has a rainbow spanning tree.

**Bibliography**


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