

This review is meant to help you on what you need to focus for the test. It is not meant to replace the book, the homework assignments, or any other activities we do in class.

1. Consider the surface  $z^2 = 3x^2 + 3y^2$ .
  - (a) Sketch the graph of the surface.
  - (b) How many points are in the surface corresponding to  $x = 3$  and  $y = 1$
  - (c) The surface correspond to the level surface of a three variable function. Which function?
  - (d) Compute the gradient of the function of part 1c.
  - (e) Compute the tangent plane to the surface  $z^2 = 3x^2 + 3y^2$  at the point  $(1, 1, \sqrt{6})$ .
2. Let  $f(x, y) = \ln(x^2 + y^2 - 9)$  be a function in two variables.
  - (a) Find the domain of  $f(x, y)$ .
  - (b) Find the range of the function  $f(x, y)$ .
  - (c) Draw the countour map of  $z = f(x, y)$ , in particular draw the level curve corresponding to the value  $z = 0$ .
  - (d) Find the tangent plane to the graph of the function at the point corresponding to  $(3, 1)$ .
  - (e) Give the equation of the linear approximation of the function at the point corresponding  $(3, 1)$ .
  - (f) Use the equation in 2e to approximate the value of the function at  $(3.001, 1.001)$ .
  - (g) Compute the gradient of the function  $f(x, y)$  at the point  $(3, 1)$ .
  - (h) Draw the gradient in the countour map you draw in 2c.
  - (i) Compute the tangent line to the level curve of the function that passes through the point  $(3, 1)$ .
  - (j) Compute the directional derivative  $D_{\mathbf{u}}(f(3, 1))$  where  $\mathbf{u}$  is the unit vector corresponding to the vector  $\langle 1, 2 \rangle$ .
  - (k) In which direction does the maximum rate of change of the function  $f$  at the point  $(3, 1)$  occur?
  - (l) What is the maximum rate of change for the function  $f(x, y)$  at the point  $(3, 1)$ .
  - (m) Describe what you obtain intersecting the graph of the function with the plane  $x = 3$ .
3. Write the equation of three different surfaces which intersections with planes parallell to the  $xy$ -plane are circles.
4. Let  $f(x, y) = \ln(x^2y) + \sin(xy + y^2)$  be a function in two variables.
  - (a) Assume that  $z = f(x, y)$ ,  $x = t^2e^{t^2}$  and  $y = t$ , compute  $\partial f(x, y)/\partial t$ .
  - (b) Assume that  $z = f(x, y)$ ,  $x = st^{-8}$  and  $y = \tan s^2$ , compute  $\partial z/\partial t$  and  $\partial z/\partial s$ .
5. Decide whether the following limits exist

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 + xy}{x^2 - y^2} \quad \text{and} \quad \lim_{(x,y) \rightarrow (0,0)} \frac{x^2 + xy}{x + y^2}$$