

Quiz 9

1. Parametrize the following curves:

- (a) The portion of the circle $x^2 + y^2 = 8$ from $(2, -2)$ to $(2, 2)$.

Solution

$$x(t) = \sqrt{8} \cos t, y(t) = \sqrt{8} \sin t \text{ and } -\pi/4 \leq t \leq \pi/4.$$

- (b) The portion of the circle $(x - 2)^2 + (y + 4)^2 = 9$ going from $(5, -4)$ to $(2, -1)$.

Solution

$$x(t) = 3 \cos(t) + 2, y(t) = 3 \sin(t) - 4 \text{ and } 0 \leq t \leq \pi/2.$$

- (c) The portion of the curve $y = 4, z = x^2 + x$ going from $(1, 4, 2)$ to $(-2, 4, 2)$.

Solution

$$x(t) = 1 - t, y(t) = 4, z(t) = (1 - t)^2 + (1 - t) \text{ and } 0 \leq t \leq 3.$$

2. Evaluate $\int_C (xy + y + z) ds$ along the curve $\mathbf{r}(t) = 2t\mathbf{i} + t\mathbf{j} + (2 - 2t)\mathbf{k}, 0 \leq t \leq 1$.

Solution

First note that $v(t) = 2\mathbf{i} + \mathbf{j} - 2\mathbf{k}$, so $|v(t)| = \sqrt{2^2 + 1^2 + (-2)^2} = 3$. So we have

$$\int_C (xy + y + z) ds = \int_0^1 ((2t)(t) + t + (2 - 2t))(3) dt = 3 \int_0^1 2t^2 - t + 2 dt = 13/2.$$

3. Find the work done by the force $\mathbf{F} = xy\mathbf{i} + (y - x)\mathbf{j}$ over the straight line from $(1, 1)$ to $(2, 3)$.

Solution

$$W = \int_0^1 (1 + t)(2t + 1)(1) dt + \int_0^1 ((2t + 1) - (t + 1))(2) dt = \int_0^1 2t^2 + 5t + 1 dt = 25/6.$$