Quiz 8

1. Set up a triple integral which gives the volume of the solid bounded below by the plane \( z = 0 \), above by the sphere \( x^2 + y^2 + z^2 = 4 \) and on the sides by the cylinder \( x^2 + y^2 = 1 \). You do not need to integrate.

**Solution** This is easiest to do in cylindrical coordinates. The \( z \) limits are 0 and \( \sqrt{4 - r^2} \). Projecting down onto the \( xy \)-plane we get a circle of radius 1. So the integral is

\[
\int_0^1 \int_0^{2\pi} \int_0^{\sqrt{4 - r^2}} r \, dz \, d\theta \, dr.
\]

2. Find the volume of the solid enclosed by the cone \( z = \sqrt{x^2 + y^2} \) and between the planes \( z = 1 \) and \( z = 2 \).

**Solution**

There are (at least) three ways to do this one. One way is in spherical coordinates, but the \( \rho \)-limits are a little messy. The other two ways are in cylindrical coordinates. I’ll do both ways.

For the first one, we integrate with respect to \( z \) first. We have to break it up into two integrals. Above the circle \( r = 1 \), \( z \) enters the region at \( z = 1 \) and leaves at \( z = 2 \). Above the region inside \( r = 2 \) and outside \( r = 1 \), \( z \) enters the region at \( z = r \) and leaves at \( z = 2 \). Thus the volume is given by

\[
\int_0^1 \int_0^{2\pi} \int_1^2 r \, dz \, d\theta \, dr + \int_1^2 \int_0^{2\pi} \int_0^2 r \, dz \, d\theta \, dr = \frac{7\pi}{3}.
\]

The second way, we integrate with respect to \( r \) first. The \( r \) limits go from 0 to \( z \). Then the \( z \) limits are 1 to 2 and the \( \theta \) limits are 0 to \( 2\pi \). We get

\[
\int_0^{2\pi} \int_1^2 \int_0^z r \, dr \, dz \, d\theta = \frac{7\pi}{3}.
\]