Quiz 1

1. Consider the vectors $\mathbf{AB}, \mathbf{AC}$ connecting the points $A = (-1, 0), B = (2, 1), C = (1, -2)$ (recall that $A$ is the tail of $\mathbf{AB}$). Find the angle between the vectors $\mathbf{AB}, \mathbf{AC}$.

   **Solution**
   
   We first find the vectors: $\mathbf{AB} = (3, 1), \mathbf{AC} = (2, -2)$. The angle between these two vectors is
   
   $$\theta = \cos^{-1} \left( \frac{\mathbf{AB} \cdot \mathbf{AC}}{|\mathbf{AB}| |\mathbf{AC}|} \right) = \cos^{-1} \left( \frac{1}{\sqrt{5}} \right)$$

2. Find the area of the triangle in $\mathbb{R}^3$ formed by the three points $P = (-2, 2, 0), Q = (0, 1, -1), R = (-1, 2, -2)$.

   **Solution**
   
   Set $\mathbf{u} = (2, -1, -1)$ the vector from $P$ to $Q$ and $\mathbf{v} = (1, 0, -2)$ the vector from $P$ to $R$. We first compute
   
   $$\mathbf{u} \times \mathbf{v} = \begin{bmatrix} i & j & k \\ 2 & -1 & -1 \\ 1 & 0 & -2 \end{bmatrix} = 2i + 3j + k.$$

   The area of the triangle is
   
   $$A = \frac{1}{2} |\mathbf{u} \times \mathbf{v}| = \frac{1}{2} (\sqrt{4 + 9 + 1}) = \frac{\sqrt{14}}{2}.$$