Mathematics 208
Fall 2008 Name ____________________

Quiz 11

1. Show that the value of $\int_C xy^2\,dx + (x^2y + 2x)\,dy$ around any square depends only on the area of the square and not on its location in the plane. (*Hint:* What does Green's Theorem say?)
Solution
By Green's Theorem we have that
$$\int_C xy^2\,dx + (x^2y+2x)\,dy = \iint_R (2xy+2) - (2xy)\,dA$$
where R is the square bounded by C. But
$$\iint_R (2xy+2) - (2xy)\,dA = 2\iint_R dA = 2(\text{area of } R).$$
Which shows that the original line integral does not depend on the position of R.

2. Consider the surface that is the portion of the plane $y + 2z = 2$ above the circle $x^2 + y^2 = 1$. Use a parameterization to express the surface area as a double integral and then evaluate the integral. (*Hint:* Parametrize the disc (i.e. solid circle) of radius 1 in the xy-plane, *not* just the circle.)
Solution
One possible parameterization for the surface is given by
$$\mathbf{r}(r,\theta) = r\cos\theta\mathbf{i} + r\sin\theta\mathbf{j} + [(2 - r\sin\theta)/2]\mathbf{k}$$
for $0 \le r \le 1$ and $0 \le \theta \le 2\pi$.
This gives partial derivatives $\mathbf{r}_\theta = (-r\sin\theta)\mathbf{i} + (r\cos\theta)\mathbf{j} + [(-r\cos\theta)/2]\mathbf{k}$ and $\mathbf{r}_r = \cos\theta\mathbf{i} + \sin\theta\mathbf{j} + (-\sin\theta)/2)\mathbf{k}$. The cross product is $\mathbf{r}_\theta \times \mathbf{r}_r = (-r/2)\mathbf{j} - r\mathbf{k}$ and thus
$$|\mathbf{r}_\theta \times \mathbf{r}_r| = \frac{\sqrt{5}r}{4}.$$
Now the surface area is given by
$$\int_0^{2\pi}\int_0^1 \frac{\sqrt{5}r}{4}\,dr\,d\theta = \frac{\sqrt{5}\pi}{4}.$$