Math 208
Gradient applications additional problems

1. Let \( f(x, y) = -0.25x^2 - 0.25y^2 \).
   a) Find \( \nabla f(-3, 4) \).
   b) Find an equation for the level curve of \( f \) going through the point \((-3,4)\).
   c) Find an equation for the tangent line at \((-3,4)\) to the equation found in (b).
   d) Find a system of parametric equations and a non-parametric equation for the normal line to the level curve of \( f \) going through the point \((-3,4)\).
   e) In which direction does \( f \) decrease most rapidly at \((-3,4)\), and what is the rate of change of \( f \) at \((-3,4)\) in that direction?
   f) What is the directional derivative of \( f \) at \((-3,4)\) in the direction from there toward (0,1)?
   g) Find all unit vectors \( \mathbf{u} \) such that \( f_{\mathbf{u}}(-3, 4) = 0 \).
   h) Find the unit vectors in all directions for which the rate of change of \( f \) at \((-3,4)\) equals 2.

2. Let \( f(x, y, z) = x^3z - 2yz^2 - 2z \). Simplify all numbers in this problem.
   a) Find \( \nabla f(2, -1, 3) \).
   b) Find the rate of change of \( f \) at \((2, -1,3)\) in the direction from there toward \((-4,1,6)\).
   c) Find an equation for the tangent plane to the level surface \( f(x, y, z) = 36 \) at the point \((2, -1,3)\).
   d) Find parametric equations for the normal line to the level surface \( f(x, y, z) = 36 \) at the point \((2, -1,3)\).
   e) Find a unit vector in the direction in which \( f \) increases most rapidly at \((2, -1,3)\), and the rate of change of \( f \) in that direction.
Gradient applications handout answers:

1. a) \(< 1.5, -2 >\) 
   b) \(-0.25x^2 - 0.25y^2 = -6.25\), or \(x^2 + y^2 = 25\)
   c) \(1.5(x + 3) - 2(y - 4) = 0\) or equivalent, such as \(4y - 3x = 25\)
   d) Parametric: \(x = -3 + 1.5t\) and \(y = 4 - 2t\)
      Non-parametric: \(\frac{x+3}{1.5} = \frac{y-4}{-2}\) or equivalent such as \(4x + 3y = 0\)
   e) Direction is direction of \(< -1.5, 2 >\), rate of change is \(-2.5\)
   f) \(\frac{3.5}{\sqrt{2}}\)
   g) \(\mathbf{u} = < 0.8, 0.6 >\) or \(\mathbf{u} = < -0.8, -0.6 >\)
   h) \(< 0, -1 >\) and \(< 0.96, -0.28 >\) are the only such unit vectors

2. a) \(<36, -18, 18> = 18<2, -1, 1>\) 
   b) \(-28 \frac{2}{7}\)
   c) \(2x - y + z = 8\) (or equivalent)
   d) \(x = 2 + 2t\), \(y = -1 - t\), \(z = 3 + t\) (or equivalent)
   e) unit vector \(\frac{<2, -1, 1>}{\sqrt{6}}\), rate of change \(18 \sqrt{6}\)