1. (15 pts) Suppose someone claimed that $\text{RL} = \text{NL}$ by the following argument:

"Proof": We shall show that Reachability for directed acyclic graphs can be decided in randomized log-space with probability of success at least one half. Given $G, s, t$, randomly walk starting at $s$, selecting an outgoing neighbor uniformly at random at each step. If the walk reaches a vertex with no outgoing neighbor, start over at $s$. If there is a path from $s$ to $t$, then eventually, the random walk should reach $t$. Therefore, there is a positive probability that the machine will accept. End "Proof"

Show this crackpot that they are wrong by demonstrating a directed acyclic graph on $n$ vertices so that there is a path from $s$ to $t$, but the expected number of random walks before reaching $t$ is $\Omega(2^n)$. Conclude this algorithm is not an $\text{RL}$-type algorithm.

2. (15 pts) Let $f$ be a function on $n = k^2$ variables that is the AND of $k$ ORs, each of disjoint $k$ variables. Prove $D(f) = n$ (where $D(f)$ is the decision tree complexity of $f$).

3. (15 pts; required for 824 students) Show that computing the permanent for matrices with integer entries is in $\text{FP}^\text{#SAT}$. Hint: Use the combinatorial interpretation of weighted cycle-covers from page 347.