1. (10pts) Suppose $L_1, L_2 \in \text{NP}$.
   (a) (5pts) Is $L_1 \cup L_2 \in \text{NP}$?
   (b) (5pts) Is $L_1 \cap L_2 \in \text{NP}$?

2. (10pts) In the CLIQUE problem, we are given an undirected graph $G$ and an integer $k$ and decide if there is a subset $S$ of $k$ vertices so that every pair $u, v \in S$ has $uv$ as an edge of $G$ (i.e. $S$ is a clique):
   \[
   \text{CLIQUE} = \{[G], [k] : \exists S \subseteq V(G), |S| \geq k, S \text{ is a clique}\}.
   \]
   Prove that CLIQUE is NP-complete.

3. (10pts) In the VERTEXCOVER problem, we are given an undirected graph $G$ and an integer $k$ and have to decide whether there is a subset $S$ of at most $k$ vertices such that for every edge $uv \in E(G)$ at least one of $u$ or $v$ is in $S$ (the $k$ vertices cover the edge set):
   \[
   \text{VERTEXCOVER} = \{[G], [k] : \exists S \subseteq V(G), |S| \leq k, \forall uv \in E(G), S \cap \{u, v\} \neq \emptyset\}.
   \]
   Prove that VERTEXCOVER is NP-complete.

4. (Required for 824 students only; 15pts) Let $\Sigma_2\text{SAT}$ denote the following decision problem: Given a quantified formula $\psi$ of the form
   \[
   \psi = \exists x \in \{0, 1\} \forall y \in \{0, 1\} \varphi(x, y) = 1,
   \]
   where $\varphi$ is a CNF formula, decide whether $\psi$ is true. That is, decide whether there exists an $x$ so that for all $y$, $\varphi(x, y)$ is true.
   Prove that if $P = NP$, then $\Sigma_2\text{SAT}$ is in $P$.  