1 Section 9.1: Polar Coordinates

What: Define points $P(r, \theta)$ defined by $x = r\cos\theta$, $y = r\sin\theta$, and $x^2 + y^2 = r^2$.
How: Simplify and reduce the given equation into forms above.
Link: http://en.wikipedia.org/wiki/Polar_coordinate_system

2 Section 9.2: Graphing in Polar Coordinates

When: Asked to draw a curve in the xy-plane, given the polar definition $r = f(\theta)$.
How: Perform symmetry tests.
Plot the curve in the θr-plane.
Plot "known" values into xy-plane.
Use derivative to find angle of curve at points.
Why? You can make some sweet-looking curves not normally possible with $y = f(x)$.

3 Section 9.3: Areas and Lengths in Polar Coordinates

3.1 Areas

When: Given $r = f(\theta)$, calculate the area in the ray between the origin and the curve.
How: Given a shape with the origin within it, compute $\int_a^b \frac{1}{2} r^2 d\theta$.
Given two curves $r_1 \geq r_2$ over $\theta \in [a, b]$, wanting to find area between them, compute $\int_a^b \frac{1}{2}(r_1^2 - r_2^2) d\theta$.
Why? Some shapes are easier to compute area in this fashion.
Tips: Sometimes, you already know the area of the shape, as in a semicircle.

3.2 Length of Polar Curve

When: Given $r = f(\theta)$, calculate the length of the curve for $a \leq \theta \leq b$.
How: Given a shape with the origin within it, compute $\int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta$.
Why? Some curves are easier to compute length in this fashion.
Tips: Sometimes, you already know the length of the curve, as in a semicircle.

4 Section 10.1: Three-Dimensional Coordinate System

What: Points in 3-space: (x_0, y_0, z_0).
How: Distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2):
$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$.
Sphere of radius a centered at (x_0, y_0, z_0): $(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = a^2$.
Link: http://en.wikipedia.org/wiki/Cartesian_coordinate_system

5 Section 10.2: Vectors

What: A vector is a direction and magnitude. Can be represented by an arrow pointing from the origin to a point.
Component form: $\mathbf{v} = \langle v_1, v_2, v_3 \rangle$
Unit Vector Form: $\mathbf{v} = v_1\mathbf{i} + v_2\mathbf{j} + v_2\mathbf{k}$.

How: There are several rules about arithmetic of vectors:
Addition: $\mathbf{u} + \mathbf{v} = \langle u_1 + v_1, u_2 + v_2, u_3 + v_3 \rangle$.
Scalar Multiplication: $j\mathbf{v} = \langle kv_1, kv_2, kv_3 \rangle$.
Length: $|\mathbf{v}| = \sqrt{v_1^2 + v_2^2 + v_3^2}$.
$\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}, \qquad (\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w}),$
$\mathbf{u} + \mathbf{0} = \mathbf{u}, \qquad \mathbf{u} + (-\mathbf{u}) = \mathbf{0},$
$0\mathbf{u} = \mathbf{0}, \qquad 1\mathbf{u} = \mathbf{u} \qquad a(b\mathbf{u}) = (ab)\mathbf{u},$
$a(\mathbf{u} + \mathbf{v}) = a\mathbf{u} + a\mathbf{v}, \qquad (a + b)\mathbf{u} = a\mathbf{u} + b\mathbf{u}.$

Why? Physics is all about the vector representation of forces, movement and location.

Link: http://en.wikipedia.org/wiki/Vector_(spatial)

6 Section 10.3: The Dot Product

What: $\mathbf{u} \cdot \mathbf{v} = u_1v_1 + u_2v_2 + u_3v_3 = |\mathbf{u}| \cdot |\mathbf{v}| \cos\theta$.
Therefore, $\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{u}|\cdot|\mathbf{v}|}$.
$\mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{u}, \qquad (c\mathbf{u}) \cdot \mathbf{v} = c(\mathbf{u} \cdot \mathbf{v}),$
$\mathbf{u} \cdot (\mathbf{v} + \mathbf{w}) = \mathbf{u} \cdot \mathbf{v} + \mathbf{u} \cdot \mathbf{w}, \qquad \mathbf{u} \cdot \mathbf{u} = |\mathbf{u}|^2, \qquad \mathbf{0} \cdot \mathbf{u} = 0.$
Component of $\mathbf{u}$ onto $\mathbf{v}$ (scalar): $\text{comp}_{\mathbf{v}}\mathbf{u} = \frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{v}|} = |\mathbf{u}| \cos\theta$.
Projection of $\mathbf{u}$ onto $\mathbf{v}$ (vector): $\text{proj}_{\mathbf{v}}\mathbf{u} = \text{comp}_{\mathbf{v}}\mathbf{u}\frac{\mathbf{v}}{|\mathbf{v}|} = \frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{v}|^2}\mathbf{v}$.

Why? A very important calculation.

Link: http://en.wikipedia.org/wiki/Dot_product

7 Section 10.5: Lines and Planes in Space

What: Use vector math to find parametric equations.
Vector equation for line through $\mathbf{r}_0$ parallel to $\mathbf{v}$:
$\mathbf{r}(t) = \mathbf{r}_0 + t\mathbf{v}, \qquad -\infty < t < +\infty.$
Parametric Equations for line through $P_0(x_0, y_0, z_0)$ parallel to $\mathbf{v} = v_1\mathbf{i} + v_2\mathbf{j} + v_3\mathbf{k}$:
$x = x_0 + tv_1, y = y_0 + tv_2, z = z_0 + tv_3, -\infty < t < +\infty.$

8 Section 11.1: Derivatives of Vector Functions

What: Given a vector function $\mathbf{r}(t)$, compute the velocity and acceleration by taking first and second derivatives coordinate-wise.
Position: $\mathbf{r}(t) := \langle f(t), g(t), h(t) \rangle$.
Velocity: $\mathbf{v}(t) = \mathbf{r}'(t) = \langle f'(t), g'(t), h'(t) \rangle$.
Acceleration: $\mathbf{a}(t) = \mathbf{v}'(t) = \langle f''(t), g''(t), h''(t) \rangle$.

Thm: If $\mathbf{r}(t)$ is a differentiable vector function of t of constant length ($|\mathbf{r}(t)| = L$ for all t), then $\mathbf{r} \cdot \frac{d\mathbf{r}}{dt} = 0$. In other words, a vector function of constant length has an orthogonal velocity vector.

9 Section 11.2: Integration of Vector Functions

What: Compute antiderivatives of vector functions given starting points.

How: Integrate coordinate-wise, and modify each constant to match the point given.

Ex: Compute $\mathbf{r}(t)$ knowing the following information:
The position at time $t_0 = 0$ is $\mathbf{r}(0) = \langle 1, 2, 3\rangle$.
The velocity at time $t_0 = 0$ is $\mathbf{v}(0) = \langle 0, 0, 4\rangle$.
The acceleration function is given by $\mathbf{a}(t) = \langle \sin(t), -\cos(t), e^t\rangle$.

Answer: Integrate $\mathbf{a}$: $\mathbf{v}(t) = \int \mathbf{a}(t)dt = \langle -\cos(t) + C_1, -\sin(t) + C_2, e^t + C_3\rangle$.
Match this to $\mathbf{v}(0) = \langle 0, 0, 4\rangle$, so $C_1 = 1, C_2 = 0, C_3 = 3$.
Thus, $\mathbf{v}(t) = \langle -\cos(t) + 1, -\sin(t), e^t + 3\rangle$.
Integrate $\mathbf{v}$: $\mathbf{r}(t) = \int \mathbf{v}(t)dt = \langle \sin(t) + t + D_1, -\cos(t) + D_2, e^t + 3t + D_3\rangle$.
Match this to $\mathbf{r}(0) + \langle 1, 2, 3\rangle$, so $D_1 = 1$, $D_2 = 3$, $D_3 = 2$.
Thus, $\mathbf{r}(t) = \langle \sin(t) + t + 1, -\cos(t) + 3, e^t + 3t + 2\rangle$.