Math 203: Contemporary Mathematics

Writing assignment 2

Assigned Thursday, January 29; due Tuesday, February 17

Choose one of the topics below.

1. In class we talked about "the" Caesar cipher, in which the letters of a message are encoded by shifting them three places forward in the alphabet (so A becomes D, B becomes E, C becomes F, ..., W becomes Z, X becomes A, Y becomes B, and Z becomes C). To decode an encoded message, you just undo the encoding by shifting the letters *back* three places.

   This idea can easily be generalized. Instead of shifting letters three places forward, we can shift them seven letters forward, for example (so A becomes H, B becomes I, C becomes J, and so on); or we could shift them two letters back (so A becomes Y, B becomes Z, C becomes A, and so on); or we could use any other shift amount. Each different shift amount will give us a different Caesar cipher. Since there are 26 letters in the alphabet, we can get 25 different Caesar ciphers (because one of the possibilities, shifting letters 26 places forward, isn't a very effective code).

   (Note that when we create a code this way and encode a message, we pick the shift amount at the beginning, but then we stick with our choice for the whole message. No fair changing shift amounts in the middle of the message—that would be a more complicated code.)

   Go to the following Web page to get a randomly selected message encoded with a Caesar cipher using a randomly chosen shift amount:

   `http://www.math.unl.edu/~s-bkell1/203-2009s/caesar.cgi`

   There are many possible messages, so everyone should get a different message.

   Can you decipher this message? Explain how you figured it out. Research the history of the Caesar cipher and the codebreaking technique called frequency analysis. How does frequency analysis work?

2. Research the Penrose tiling. This is a tiling of the (infinite) plane using a set of special tiles. A Penrose tiling is interesting because, unlike many ways of tiling the plane, it does not have translation symmetry. Describe the rules for fitting the tiles together, and give some interesting properties of the Penrose tiling. Cut out a bunch of Penrose tiles (about 50 of them at least) and make a tiling according to the rules; use this to illustrate some of the interesting properties you described.

3. Find an artist who uses tilings or symmetry in his or her work, or a style of art in which these things are prevalent. Some examples include Islamic art and architecture, pottery, mosaics, quilts and textiles, and the work of M. C. Escher. Describe the mathematics behind this art. What do you think makes this art visually appealing?