Math 203: Contemporary Mathematics
Project (200 points)
Assigned Tuesday, March 31; due Thursday, April 23

This assignment is a group project. You will work in a group of up to four people (i.e., the maximum group size is four). The project is due on Thursday, April 23, 2009.

Your assignment is to address one of the projects outlined here, and write up your report in a paper. All group members should sign the final report (one report for the whole group). When you sign, you are certifying that you made a fair and honest contribution to the group and that you understand what has been written. I expect to be told of anyone trying to get a free ride. All group members will receive the same grade.

You do not necessarily need to address every one of the questions in the project you choose. Conversely, you are welcome to consider additional questions besides the ones listed here. The most important thing is to think critically, creatively, and mathematically about the topic of the project, and to cover it in sufficient depth and detail. Have fun with it!

A paper of sufficient length will probably be 6 to 8 pages long (not counting pictures, diagrams, tables, etc.), though longer papers are certainly fine.

Note that correct grammar, spelling, readability, and mathematical content all count. Any graphs, diagrams, or other illustrations should be neat, accurate, and appropriately labeled. Start with an introductory paragraph, organize your report in a sensible way, walk your reader through what you are doing, and end with a conclusion. You should not simply number your answers 1, 2, 3, ...; you are writing a report, so the exposition of your ideas should flow smoothly from the beginning to the end. Use transition sentences where appropriate. Your paper should make sense to someone seeing it for the first time, without reference to the project handout.

Part of the point of a group project like this is to work together as a team. This includes dealing with the logistical problems of finding time in your schedules, determining the division of labor, and dealing with personality differences. I hope you can resolve any problems yourselves, but if you have problems that you cannot resolve, please see me. Also, if you have a member who is not contributing, the group has a right to ask that person to leave the group. However, please keep me informed of any changes in your group membership.

I'll be happy to meet with your group to discuss things, subject to the following conditions:

(a) All of your group members must be present at the meeting, and
(b) You must come to the meeting prepared (this means, for example, that you should spend time thinking about things before coming to talk to me).

Be prepared to explain your ideas and to ask specific questions. I'll also be happy to meet with your group to review a draft of your report and make comments and suggestions.

Remember the quality expectations for submitted work in the course information sheet. These are especially important for the project.

- **Be academically honest.** Never put your name on work that is not yours or work you do not understand, and always give credit where credit is due. Remember: Taking someone else's work or ideas and presenting them as your own is plagiarism, even if you change the wording. Provide a list of sources other than the textbook (if any) that you used to do the assignment, and state clearly that you are copying or mimicking an example from the book in order to do the assignment (if appropriate). If you worked with other students in the class to solve a problem, be sure to write your results *in your own words*, and include a list of your collaborators. Do not ever copy text directly from another source, even with attribution, unless it is a short quote properly marked as such.

- Put your names (first and last) in the top right-hand corner of every page you turn in. Do not include your student ID numbers or your Social Security numbers. On the first page, include a short description of the project (a few words as a title would be fine—just so I know what it is I'm looking at).
- Type your project (except perhaps for figures).
- Double-space your paper, and leave wide margins (1.5 inches all around is nice), so that I have enough blank space on each page to write comments.
- Use a standard font, and print text in black.
- Use good quality paper, print on only one side, and staple the pages together.
- Include page numbers, in case the pages are accidentally stapled in the wrong order.

Project 1: All About Primes

A prime number is a positive integer that is divisible only by itself and 1 (but usually the number 1 itself is not considered to be prime). These numbers are considered the building blocks of all mathematics, because any integer can be decomposed into a unique product of primes, and they have been studied extensively beginning with the ancient Greeks. However, there are still many fundamental properties that are unknown. For this project, you will be researching the prime numbers: lots of things that we know, but also many things we think are true but that are still conjectures.

1. Explain the general background for prime numbers. What is a prime number? What is the Fundamental Theorem of Arithmetic? How can you find primes using the Sieve of Eratosthenes? Is this a good method? What is the history of prime numbers? Who are some of the mathematicians who have studied them and made important contributions to the area? And finally, why are prime numbers so important?

2. Search for Euclid's proof that there are infinitely many primes, and explain it. (Come talk to me if you need help understanding it.)

In the next few steps, you'll discover the ways you can make money from prime numbers!

3. Look up the Great Internet Mersenne Prime Search and the Electronic Frontier Foundation's Cooperative Computing Awards. Explain what both are, why this contest is being held, and the recent discoveries. (The search for prime numbers closely mirrors the history of computing. For a timeline of record-breaking prime numbers through the years, see `http://primes.utm.edu/notes/by_year.html`.)

4. Research the RSA public key system. (There was money to be made here, but the RSA Factoring Challenge was recently discontinued. A good page describing the contest and the factoring of the number called "RSA-640" is `http://mathworld.wolfram.com/news/2005-11-08/rsa-640/`.) Completely answer the questions in problem 48 from Section 1.3 in the textbook. (For the example, just use smallish primes.)

5. Look into the Riemann Hypothesis and the Clay Mathematics Institute's prize for that. (It will be hard to explain exactly what the hypothesis says, but write about the possible implications of a proof of it.) Here is a good article from the New York Times to get you started: `http://www.nytimes.com/2002/07/02/science/143-year-old-problem-still-has-mathematicians-guessing.html`.

6. Explain the Goldbach Conjecture and the Twin Prime Conjecture. For the Twin Prime Conjecture, you might start with the following page: `http://www.pbs.org/wgbh/nova/sciencenow/3302/02-numb-flash.html`.

Project 2: Statistical Study

For this project, you will pose a question to be answered, conduct research to gather data, and analyze your results statistically to reach a conclusion.

1. You must create some sort of statistical study. I want your study to have quantitative data so that you can do the analysis mentioned in Part 4. This means that you must come up with some hypothesis or question that you wish to try to answer based on statistical data. Think about how you are going to do this. Are you going to do an experiment, something observational, a survey, or what?
    - State your hypothesis or question and the goal of the study.
    - Decide what type of study you will do (an observation, an experiment, a survey, etc.) and design the procedure involved in completing the task.
    - Approve your study with me before you begin.
    - Execute your plan and gather data.
    - Explain that you are conducting a survey for this class in any survey that you hand out or experiments that you conduct with people.

2. Now you must analyze the data you have collected.
    - Describe your population. What sample did you select for your study? Explain why you believe you have chosen a representative sample. Describe the sampling method you used and why you feel that this was a good method for your study.
    - If you have a treatment group and a control group, describe them and how they were used in the study.
    - Describe any bias the study may have. If your study has any bias, how could it be reduced or removed if you were to conduct the study again?

3. Arrange the data in an organized way.
    - Create a frequency table or histogram for your data.
    - Create some sort of table or graph of your choice to represent your data.
    - If your study is designed to determine whether two things are related, what type of correlation do you have? Is it possible the correlation is caused by coincidence?

4. Crunch some numbers.
    - Find the range, mean, median, and mode for your data. Which of these "averages" best represents your data? Do you have any outliers?
    - Give the five-number summary for your data, and draw a box-and-whiskers plot.
    - Based on your frequency table or histogram, how many peaks does the distribution of data have? Is there symmetry or skewing?
    - Does your data look like any of the distributions we have talked about in class?

5. Make some conclusions.
    - Did your study answer your initial hypothesis or question? Was the answer what you expected it to be?
    - Do the results of your study suggest any avenues for further research? Are there questions that have not been satisfactorily answered? Have new questions been raised?

Project 3: Music and Math

Research some of the relationships between music and mathematics. Summarize your findings, and explain them on a level that your classmates would understand. Here are some suggestions:

1. Explain the ideas of frequency and pitch. How are they related? How does the concept of an octave relate to frequency? What is a semitone (half step), in terms of frequency? Give some examples.
2. Explain the concept of a musical scale. Explain what diatonic and chromatic scales are.
3. Explain tuning systems, and the differences between Pythagorean tuning, just intonation, and equal temperament.
4. Explain intervals, chords, and key signatures. How do these compare in various tuning systems? What makes a chord sound harmonious or dissonant? Why are musical works written in different keys?
5. Describe the concept of time signatures. In what ways are time signatures similar to fractions? In what ways are they different?
6. Research ways in which mathematical tools or ideas have been used in the composition of music, or ways in which computer programs have been written to automatically generate music. (Some composers who have experimented with such ideas include Raymond Scott, Iannis Xenakis, Lejaren Hiller, and David Cope; you might also try looking for information on *stochastic music*, *computer music*, and *algorithmic composition*.)
7. Explain how music is recorded on a compact disc and then played again by a CD player.
8. Find other connections between mathematics and music. Report on your findings.
9. Include diagrams or musical excerpts to help illustrate the concepts that you are explaining.

Project 4: Data Compression

In this project, you will research some of the ideas behind data compression, which is used to shrink the size of computer files. Data compression is used in ZIP files, MP3s, JPEG images, DVDs, and streaming video (such as YouTube), for example.

1. Give an overview of what data compression is and summarize some of its history. Describe examples of its applications (and why it is important in these applications).
2. Is it possible to have a process for data compression that will always reduce the size of a file? Why or why not?
3. Explain the difference between *lossless compression* and *lossy compression*. Give examples of situations in which one of these is better than the other, and explain why.
4. Research the data compression scheme called *run-length encoding*. Describe how it works, and explain why run-length encoding is able to reduce the number of characters needed to encode a message.
5. Work through an example of using run-length encoding to encode a message. Work through an example of decoding a different message that has been encoded with run-length encoding. Explain what you are doing, and why.
6. Research the data compression scheme called *Huffman coding*. Describe how it works, and explain why Huffman coding is able to reduce the number of bits needed to encode a message. (If you need help understanding Huffman coding, please come talk to me.)
7. Work through an example of using Huffman coding to encode a message. Work through an example of decoding a different message that has been encoded with Huffman coding. Explain what you are doing, and why.

Project 5: Literature and Math

Investigate some of the relationships between literature and mathematics. Here are some suggestions:

1. Describe some of the mathematical features in the structure of poetry. What kinds of mathematical patterns appear in the meter, rhyme schemes, and stanza forms of poetry? Are there poets that have made specific attempts to incorporate mathematical concepts into their poetic structure?
2. Research the idea of constrained writing. Give some examples. Explain the patterns that are used (or avoided) in these examples.
3. Research the concepts of *stylometry* and *authorship analysis*. What are these used for? What are some of the techniques used? Give some concrete examples.
4. Research the idea of Markov chains and their application to the automatic generation of text by computers. Explain the main ideas behind Markov chains in a way your classmates could understand. Find a Markov-chain text generator online, generate some text, and explain what is happening. Does the generated text make sense? Why or why not? (If you need help understanding Markov chains, please come talk to me.)
5. Read a short story based upon a mathematical theme and comment on it. Explain the mathematics in the story and how it is used in the plot. (Three good collections of mathematically-oriented short stories are *Fantasia Mathematica*, by Clifton Fadiman, and its sequel *The Mathematical Magpie*; and *Imaginary Numbers*, by William Frucht. I've reserved these books at the Love Library circulation desk, and I have copies of them as well.)

Project 6: ???

You are welcome to propose a different project for your group to tackle. Come talk to me about it before you begin serious research. In your project proposal, explain the topic you want to research, the questions you will attempt to answer, and some of the ways in which your project relates to ideas we have discussed in class. (Your project proposal does not have to be a written document, but spend some time thinking about these things before coming to talk to me. If you have an idea for a general topic but are having difficulty finding ways in which it relates to mathematics, feel free to ask me about it.)