Math 203: Contemporary Mathematics

Chapter 6 review questions

Question 1. Describe what the following terms mean.

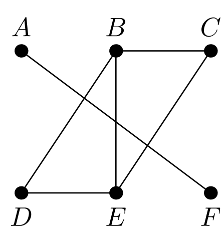
(a) graph
(b) vertex
(c) edge
(d) loop
(e) adjacent vertices
(f) degree of a vertex
(g) path
(h) circuit
(i) Eulerian path
(j) Eulerian circuit
(k) connected graph
(l) disconnected graph
(m) components of a graph
(n) bridge
(o) weighted graph
(p) weight of an edge
(q) subgraph
(r) tree
(s) spanning tree
(t) minimal spanning tree
(u) Eulerization
(v) Hamiltonian path
(w) Hamiltonian circuit
(x) complete graph
(y) complete weighted graph
(z) cost of a path
($\star$) approximation algorithm

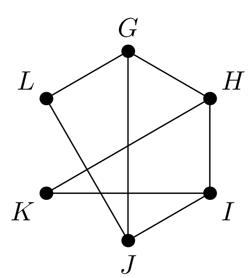
Question 2.

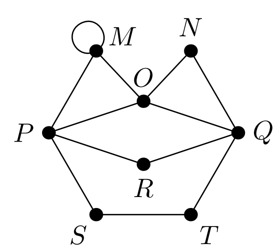
(a) What are the differences between an edge, a loop, a bridge, a path, and a circuit?
(b) What are the differences between an Eulerian path, an Eulerian circuit, a Hamiltonian path, and a Hamiltonian circuit?
(c) When is a graph a tree?
(d) When is a subgraph a spanning tree?
(e) When is a graph a complete graph?

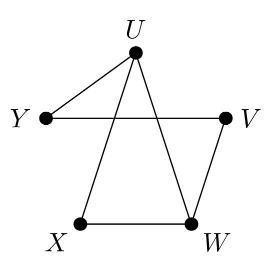
Question 3. For each of the five graphs below, answer the following questions.

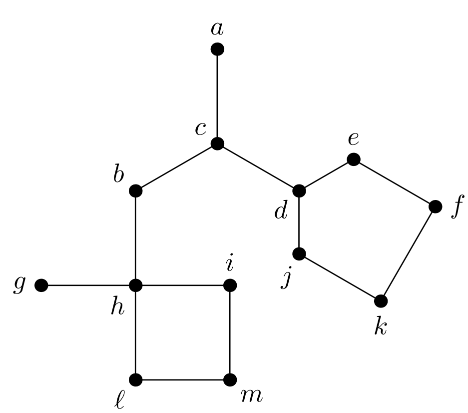
(a) How many vertices does the graph have? How many edges does it have? How many loops does it have? How many circuits does it have?
(b) Is the graph connected or disconnected? If the graph is connected, how many bridges does it have? (It might have no bridges.) If the graph is disconnected, how many components does it have?
(c) Does the graph have an Eulerian path or an Eulerian circuit? If so, describe it. If not, why not?
(d) Does the graph have a Hamiltonian path or a Hamiltonian circuit? If so, describe it. If not, why not?







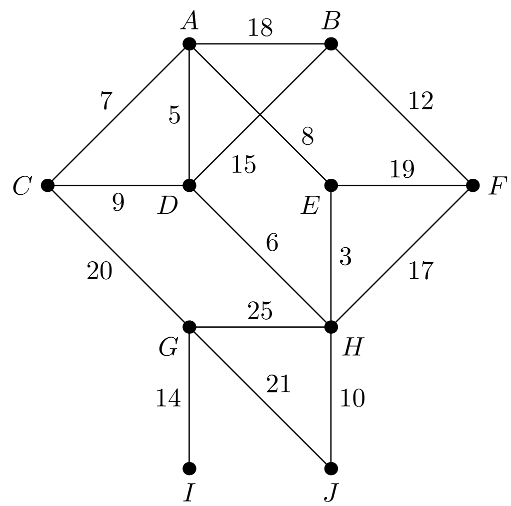




Question 4. A description of Kruskal's algorithm is given below. (There is a somewhat different description in your textbook on pages 366–368.) Use Kruskal's algorithm to find a minimal spanning tree in the weighted graph below.

Kruskal's algorithm, for finding a minimal spanning tree in a weighted graph:

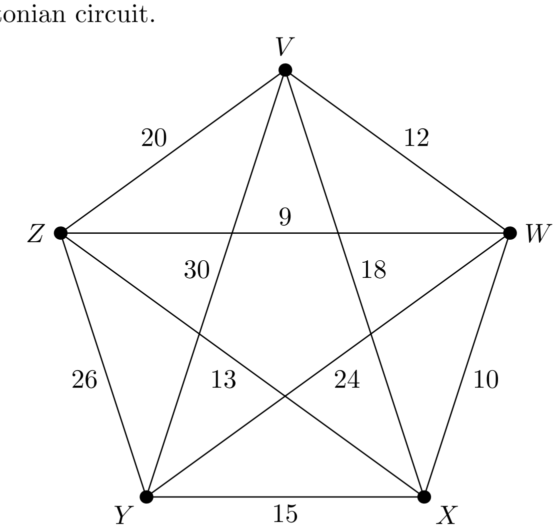
1. Begin your drawing of the minimal spanning tree by drawing only the vertices of the weighted graph.
2. Find the edge with the smallest weight, and draw that edge in your subgraph.
3. Find the edge with the next smallest weight, and draw that edge in your subgraph, **unless** that edge would complete a circuit in your subgraph.
4. Is your subgraph a connected graph? If so, you have a minimal spanning tree. If not, repeat step 3.



Question 5. Use the nearest-neighbor algorithm (described briefly below) or the cheapest-link algorithm (described in the textbook on page 389) to approximate a least-cost Hamiltonian circuit for the weighted graph below.

Nearest-neighbor algorithm, for finding a low-cost Hamiltonian circuit in a weighted graph:

1. Specify a starting vertex.
2. If unvisited vertices remain, go from the current vertex to the unused vertex that gives the least-cost connecting edge.
3. If no unvisited vertex remains, return to the starting vertex to finish forming the low-cost Hamiltonian circuit.



Question 6. A table is given below of the driving distances (in miles) between 12 Nebraska cities, according to Google Maps. A political candidate wants to visit each one of these cities to campaign. Find an efficient route for her to take. Explain the method you are using, in terms of the things we discussed in Chapter 6.

	Chad.	Col.	Frem.	G. I.	Linc.	Norf.	N. P.	Om.	Scot.	S. S. Cy.	Thed.	Val.
Chadron	—	368	399	327	421	324	229	436	105	371	178	138
Columbus	368	—	47	66	77	46	209	86	385	118	200	231
Fremont	399	47	—	113	53	76	257	38	432	73	249	262
Grand Island	327	66	113	—	96	111	144	138	320	183	149	211
Lincoln	421	77	53	96	—	129	224	59	400	126	243	314
Norfolk	324	46	76	111	129	—	254	113	381	81	205	187
North Platte	229	209	257	144	224	254	—	281	179	363	66	131
Omaha	436	86	38	138	59	113	281	—	456	96	285	298
Scottsbluff	105	385	432	320	400	381	179	456	—	477	177	244
South Sioux City	371	118	73	183	126	81	363	96	477	—	289	234
Thedford	178	200	249	149	243	205	66	285	177	289	—	66
Valentine	138	231	262	211	314	187	131	298	244	234	66	—

Question 7. A small business has five employees who each have a computer. The computers need to be connected by a network. It is not necessary that every two computers be connected directly, but there needs to be some path for information to take between any two computers. The distances (in feet) between the employees' desks are shown in the following table.

	Adam	Brenda	Cathy	Dave	Edward
Adam	—	33	46	33	31
Brenda	33	—	30	27	39
Cathy	46	30	—	14	26
Dave	33	27	14	—	14
Edward	31	39	26	14	—

(a) Draw a weighted graph representing this office.

(b) Find a way to connect the computers together using the minimum length of cable. Explain the method you are using, in terms of the things we discussed in Chapter 6.

Question 8. Recall that the sum of the degrees of all the vertices in a graph equals twice the number of edges in the graph, because the sum of all the degrees is the total number of edge-ends, and every edge has two ends. So the sum of all the degrees must be even (because it's equal to twice another number).

Use this fact to answer the following question: Is it possible to draw a graph that has exactly five vertices with odd degrees (and possibly some other vertices that have even degrees)? If so, draw an example. If not, explain why not.