1. Given the points $A=(1,1,-2)$, $B=(-1,1,1)$, and $C=(2,0,2)$:

(a) Find the distance between A and C.

$$\sqrt{(1-2)^2+(1-0)^2+(-2-2)^2} = \sqrt{1+1+16} = \boxed{\sqrt{18}}$$

(b) Find the vectors $\overrightarrow{AB}$ and $\overrightarrow{BC}$.

$$\overrightarrow{AB} = \langle -1-1,\ 1-1,\ 1-(-2)\rangle = \boxed{\langle -2, 0, 3\rangle}$$

$$\overrightarrow{BC} = \langle 2-(-1),\ 0-1,\ 2-1\rangle = \boxed{\langle 3, -1, 1\rangle}$$

(c) Find $\overrightarrow{AB} \cdot \overrightarrow{BC}$.

$$\overrightarrow{AB} \cdot \overrightarrow{BC} = (-2)\cdot 3 + 0\cdot(-1) + 3\cdot 1 = \boxed{-3}$$

(d) Find the angle between $\overrightarrow{AB}$ and $\overrightarrow{BC}$.

$$|\overrightarrow{AB}| = \sqrt{(-2)^2+0^2+3^2} = \sqrt{4+0+9} = \sqrt{13}$$

$$|\overrightarrow{BC}| = \sqrt{3^2+(-1)^2+1^2} = \sqrt{9+1+1} = \sqrt{11}$$

$$\theta = \cos^{-1}\left(\frac{\overrightarrow{AB}\cdot\overrightarrow{BC}}{|\overrightarrow{AB}|\,|\overrightarrow{BC}|}\right) = \boxed{\cos^{-1}\left(\frac{-3}{\sqrt{13}\sqrt{11}}\right)}$$

1. (e) Find an equation for the line parallel to $\overrightarrow{AB}$ which passes through the point C.

$$\boxed{\vec{r}(t) = \langle 2, 0, 2\rangle + t\langle -2, 0, 3\rangle, \quad -\infty < t < \infty}$$

or: $\vec{r}(t) = \langle 2-2t,\ 0,\ 2+3t\rangle$

2. Use a single equation or a pair of equations to describe the circle of radius 3 centered at $(7, -4, 2)$ parallel to the xz-plane.

It's parallel to the xz-plane, so $y =$ some constant, and the equation for the circle should use x's and z's. So:

$$(x-7)^2 + (z-2)^2 = 3^2$$

and

$$y = -4$$

3. Identify which of the following equations or sets of equations define a line, a point, a plane, or a sphere. If an equation or set of equations defines a sphere, identify the center and the radius of the sphere.

(a) $x = 2, \quad y = 3$

Here x has to be 2 and y has to be 3, but z can be anything. So this set of equations describes a line parallel to the z-axis which passes through the point $(2,3,0)$.

(b) $x^2 + y^2 + z^2 - 2z = 1$

Completing the square for the z terms, we get

$$x^2 + y^2 + z^2 - 2z + 1 = 1 + 1 = 2$$

so $$(x-0)^2 + (y-0)^2 + (z-1)^2 = (\sqrt{2})^2.$$

Therefore this equation defines a sphere of radius $\sqrt{2}$ centered at $(0,0,1)$.

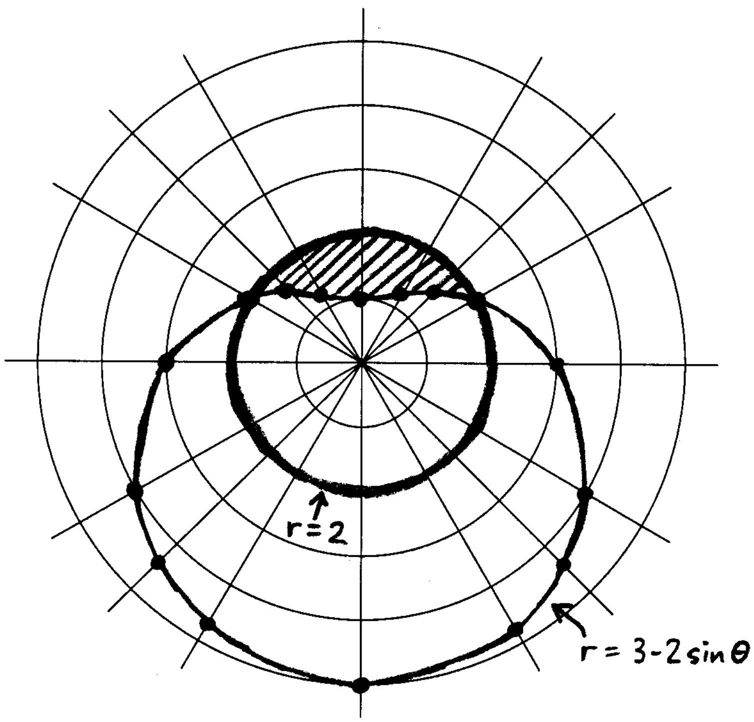
3\. (c) $x = 2, \quad y = 3, \quad z = 4$

There is only one point which satisfies all three of these equations; so this set of equations defines the point (2, 3, 4).

(d) $2x - 3y + z\sqrt{10} = -19.2$

This is an equation of the form $ax + by + cz = d$, which is the general form of the equation for a plane.

4\. (a) Sketch the polar curves $r = 2$ and $r = 3 - 2\sin\theta$ on the same polar plane.



θ	$3 - 2\sin\theta$
0	3
$\pi/6$	2
$\pi/4$	$3-\sqrt{2} \approx 1.6$
$\pi/3$	$3-\sqrt{3} \approx 1.3$
$\pi/2$	1
$2\pi/3$	$3-\sqrt{3} \approx 1.3$
$3\pi/4$	$3-\sqrt{2} \approx 1.6$
$5\pi/6$	2
π	3
$7\pi/6$	4
$5\pi/4$	$3+\sqrt{2} \approx 4.4$
$4\pi/3$	$3+\sqrt{3} \approx 4.7$
$3\pi/2$	5
$5\pi/3$	$3+\sqrt{3} \approx 4.7$
$7\pi/4$	$3+\sqrt{2} \approx 4.4$
$11\pi/6$	4

4.(b) Shade the region that lies inside $r=2$ but outside of $r=3-2\sin\theta$. Set up, but <u>do not evaluate</u>, an integral which would find the area of this shaded region. Be sure to include your limits of integration.

The shaded region lies between $\theta=\frac{\pi}{6}$ and $\theta=\frac{5\pi}{6}$, because $r=2$ for both curves at these values of θ. To find the area of the shaded region, we can find the area enclosed by the outer curve $(r=2)$ and subtract the area enclosed by the inner curve $(r=3-2\sin\theta)$.

Area enclosed by $r=2$: $\int_{\pi/6}^{5\pi/6}\frac{1}{2}(2)^2\,d\theta=\int_{\pi/6}^{5\pi/6}2\,d\theta$

Area enclosed by $r=3-2\sin\theta$: $\int_{\pi/6}^{5\pi/6}\frac{1}{2}(3-2\sin\theta)^2\,d\theta$

So the area of the shaded region is

$$\int_{\pi/6}^{5\pi/6}2\,d\theta-\int_{\pi/6}^{5\pi/6}\frac{1}{2}(3-2\sin\theta)^2\,d\theta$$

$$=\boxed{\int_{\pi/6}^{5\pi/6}\left[2-\frac{1}{2}(3-2\sin\theta)^2\right]d\theta.}$$

5. Consider again the polar curve $r = 3 - 2\sin\theta$. Set up, but <u>do not evaluate</u>, an integral which gives the length of this curve. Be sure to include your limits of integration.

As θ goes from 0 to 2π, we trace out the curve exactly once, so the length of the curve is

$$\int_0^{2\pi} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

$$= \boxed{\int_0^{2\pi} \sqrt{(3-2\sin\theta)^2 + (-2\cos\theta)^2}\, d\theta.}$$

6. Write an equivalent Cartesian equation for the polar equation $r = 5\cos\theta + 5\sin\theta$, and use this to describe or identify the graph.

Multiplying both sides by r, we get

$$r^2 = 5r\cos\theta + 5r\sin\theta.$$

Now $r^2 = x^2 + y^2$, $x = r\cos\theta$, and $y = r\sin\theta$, so we have

$$x^2 + y^2 = 5x + 5y.$$

Rearranging and completing the square, we get

$$x^2 - 5x + \left(\tfrac{5}{2}\right)^2 + y^2 - 5y + \left(\tfrac{5}{2}\right)^2 = 2\left(\tfrac{5}{2}\right)^2,$$

so $$\left(x - \tfrac{5}{2}\right)^2 + \left(y - \tfrac{5}{2}\right)^2 = \frac{25}{2} = \left(\tfrac{5}{\sqrt{2}}\right)^2.$$

So this is a circle of radius $5/\sqrt{2}$ centered at $(5/2, 5/2)$.

7. Let $\vec{s}(t)$ be the vector-valued function given by $\langle \sin(3t), \cos(3t), e^t \rangle$.

(a) [Skipped]

(b) Find $\vec{s}\,'(t)$.

Just find the derivative of each component:

$$\boxed{\vec{s}\,'(t) = \langle 3\cos(3t),\ -3\sin(3t),\ e^t \rangle.}$$

(c) Are $\vec{s}(t)$ and $\vec{s}\,'(t)$ perpendicular for some value of t? If so, find all such values of t.

Two nonzero vectors are orthogonal (perpendicular) if and only if their dot product is 0. So find $\vec{s}(t) \cdot \vec{s}\,'(t)$:

$$\begin{aligned}
&\langle \sin(3t), \cos(3t), e^t \rangle \cdot \langle 3\cos(3t), -3\sin(3t), e^t \rangle \\
&= 3\sin(3t)\cos(3t) - 3\cos(3t)\sin(3t) + e^{2t} \\
&= e^{2t}. \qquad \text{[first two terms cancel]}
\end{aligned}$$

Now e^{2t} is never 0, so $\vec{s}(t) \cdot \vec{s}\,'(t)$ is never 0, so $\vec{s}(t)$ and $\vec{s}\,'(t)$ are never perpendicular.

8. Let $\vec{r}(t) = \left\langle \frac{t^2-4}{t+2},\ t+1,\ t^2 \right\rangle$.

(a) Compute the value of $\vec{r}(t)$ at $t=0$.

$$\vec{r}(0) = \left\langle \frac{0^2-4}{0+2},\ 0+1,\ 0^2 \right\rangle = \boxed{\langle -2, 1, 0 \rangle .}$$

(b) What is the domain of $\vec{r}(t)$?

All values of t are OK except $t=-2$ (because that would make the denominator 0 in the first component). So the domain of $\vec{r}(t)$ is all real numbers except -2.

(c) For which value of a is the following vector-valued function continuous at $t=-2$?

$$\vec{s}(t) = \begin{cases} \langle a, -1, 4 \rangle, & \text{if } t=-2; \\ \vec{r}(t), & \text{if } t \neq -2 \end{cases}$$

Just like a real-valued function, a vector-valued function is continuous at a point when the value of the function at that point is equal to the limit of the function at that point. At $t=-2$, the value of the function is $\langle a, -1, 4 \rangle$.

(continued $\rightarrow$)

8. (c) — continued.

We want this to equal the limit of $\vec{s}(t)$ as $t \to -2$, which is

$$\begin{aligned}
\lim_{t\to -2} \vec{s}(t) &= \lim_{t\to -2} \vec{r}(t) \\
&= \lim_{t\to -2} \left\langle \frac{t^2-4}{t+2},\ t+1,\ t^2 \right\rangle \\
&= \left\langle \lim_{t\to -2} \frac{t^2-4}{t+2},\ \lim_{t\to -2}(t+1),\ \lim_{t\to -2} t^2 \right\rangle \\
&= \left\langle \lim_{t\to -2} \frac{(t+2)(t-2)}{t+2},\ -1,\ 4 \right\rangle \\
&= \left\langle \lim_{t\to -2} (t-2),\ -1,\ 4 \right\rangle \\
&= \langle -4, -1, 4 \rangle.
\end{aligned}$$

So we want $\langle a, -1, 4 \rangle = \langle -4, -1, 4 \rangle$, so clearly we must have $\boxed{a = -4.}$