Math Excel 106 **Worksheet 1** Fall 2007 - Tuesday, 08/28/2007

1. Below is a list of "simple" algebra problems. Some of the solutions are correct and some are not. For each problem, determine whether the answer is correct. Then, list any mistakes in the steps of the solutions. Note that there can still be mistakes in the solution even if the final answer is correct.

   (a) $\dfrac{x^2-1}{x+1} = \dfrac{x^2+(-1)}{x+1} = \dfrac{x^2}{x} + \dfrac{-1}{1} = x - 1$

   (b) $(x+y)^2 = x^2 + y^2$

   (c) $(\sqrt{x})^2 = x = \sqrt{x^2}$

   (d) $\log(ab) = \log a + \log b$

   (e) $e^{x-y} = e^x e^y$; $\quad e^{rx} = (e^x)^r$

   (f) $\dfrac{x^2y^5}{2x^{-3}} = x^2y^5 \cdot 2x^3 = 2x^6y^5$

   (g) $\dfrac{x^{-1}+y^{-1}}{x^{-1}-y^{-1}} = \dfrac{(x+y)^{-1}}{(x-y)^{-1}} = \left(\dfrac{x+y}{x-y}\right)^{-1} = -\dfrac{x+y}{x-y} = \dfrac{x+y}{y-x}$

   (h) $\sqrt{x^2+y^2} = x + y$

   (i) $10^{\ln a} = a$

   (j) $\dfrac{\log a}{\log b} = \log(a-b)$

2. Solve the following inequalities. Whenever possible, use distance arguments.

   (a) $|x-3| < 5$ (b) $-1 \le |x+1| \le 2$ (c) $|x^2+2| < |x^2+1|$ (d) $x^4 - x \le 0$

   (e) $(x-4)(x+1) < 0$ (f) $(x-4)(x+1) < -6$ (g) $x^2 < 0$ (h) $x^2 < 1$

3. You take a turkey out of the oven and let it cool before serving it to your family. The temperature in degrees Fahrenheit of the turkey t minutes after taking it out of the oven is given by the function

$$g(x) = 110e^{-0.0256t} + 70\ .$$

   Your grandfather declares that he will not eat the turkey unless its temperature is at least 120 degrees, but Aunt Beatrice will refuse to eat the turkey unless its temperature is below 150 degrees. During what time interval should you serve the turkey so that both Grandpa and Aunt Beatrice will eat it?

4. True or False: If $x < 0$, then $|x| = -x$. If false, give an example that illustrates your conclusion. If true, explain why. *Remember that giving an example that works doesn't prove that it's true in general.*

5. (a) Show that the expression $\dfrac{m+n+|m-n|}{2}$ is always equal to the larger of the two numbers m and n.

   (b) Write a similar expression for the smaller of the two numbers m and n.

6. (a) Suppose you graph your sister's height each year on her birthday. Will the graph pass the vertical line test? Will the graph pass the horizontal line test? Explain.

   (b) A calculus student would like to record the temperature for the fall semester. On his graph, the vertical axis corresponds to the days and the horizontal axis reflects temperature at 9:30am. Will his graph pass the vertical line test? Explain.

   (c) Now give examples of data that would (1) pass and (2) fail the vertical line test.

7. (a) Use the definition of logarithms to evaluate the following without using a calculator.

   (i) $\log_4 64$ (ii) $\log_3 \frac{1}{81}$ (iii) $\log_9 3$ (iv) $\log_\pi 1$

   (b) Write the expression $\ln b + k \ln x - 3 \ln y$ as a single logarithm.

8. Solve for x.

   (a) $2^{x+3} = 8$ (b) $\log(x-2) - \log(2x+3) = 0$

9. Estimate the slope of the graph $y = \cos x$ at $x = \pi$. How accurate do you think your estimate is? Why? What do you think the exact value is?

10. The Sierpinski triangle is an example of a fractal. A fractal is a geometric shape that is recursively constructed or self-similar, that is, a shape that appears similar at all scales of magnification and is therefore often referred to as "infinitely complex."

    To construct the Sierpinski triangle, start with an equilateral triangle with sides of length 1. Mark the midpoint of each side, and connect these midpoints to form a middle "upside-down" triangle. Remove the middle triangle, and repeat this process on the three remaining triangles. Continue repeating this process indefinitely on the triangles created in the previous step.

    (a) Let a_0 denote the area of the original equilateral triangle. Show that $a_0 = \dfrac{\sqrt{3}}{4}$.

    (b) Let a_1 be the area after the first middle triangle has been removed. Show that $a_1 = \dfrac{3\sqrt{3}}{16}$.

    (c) Continuing in the same manner, find a_2, a_3 and a_4. What do you think $\lim_{n\to\infty} a_n$ is?

    (d) Can you find a formula for computing any a_n from earlier a_n's ?