

NAME: [Solutions]

MATH 107 Quiz 2 (solutions)  
January 19, 2007

Evaluate the following integrals.

1.  $\int x^3 \cos 2x \, dx$

**Solution.** We will use integration by parts three times to evaluate this integral.

$$\begin{aligned} & \int x^3 \cos 2x \, dx \\ & \left[ \begin{array}{ll} u = x^3 & dv = \cos 2x \, dx \\ du = 3x^2 \, dx & v = \frac{1}{2} \sin 2x \end{array} \right] \\ & = \frac{1}{2} x^3 \sin 2x - \frac{3}{2} \int x^2 \sin 2x \, dx \\ & \left[ \begin{array}{ll} u = x^2 & dv = \sin 2x \, dx \\ du = 2x \, dx & v = -\frac{1}{2} \cos 2x \end{array} \right] \\ & = \frac{1}{2} x^3 \sin 2x - \frac{3}{2} \left( -\frac{1}{2} x^2 \cos 2x + \int x \cos 2x \, dx \right) \\ & = \frac{1}{2} x^3 \sin 2x + \frac{3}{4} x^2 \cos 2x - \frac{3}{2} \int x \cos 2x \, dx \\ & \left[ \begin{array}{ll} u = x & dv = \cos 2x \, dx \\ du = dx & v = \frac{1}{2} \sin 2x \end{array} \right] \\ & = \frac{1}{2} x^3 \sin 2x + \frac{3}{4} x^2 \cos 2x - \frac{3}{2} \left( \frac{1}{2} x \sin 2x - \frac{1}{2} \int \sin 2x \, dx \right) \\ & = \frac{1}{2} x^3 \sin 2x + \frac{3}{4} x^2 \cos 2x - \frac{3}{4} x \sin 2x + \frac{3}{4} \left( -\frac{1}{2} \cos 2x \right) + C \\ & = \left( \frac{1}{2} x^3 - \frac{3}{4} x \right) \sin 2x + \left( \frac{3}{4} x^2 - \frac{3}{8} \right) \cos 2x + C. \end{aligned}$$

$$2. \int 45 \sin^3(3\theta) \cos^2(3\theta) d\theta$$

**Solution.**

$$\begin{aligned} \int 45 \sin^3(3\theta) \cos^2(3\theta) d\theta &= 45 \int \sin^2(3\theta) \cos^2(3\theta) \sin(3\theta) d\theta \\ &= 45 \int (1 - \cos^2(3\theta)) \cos^2(3\theta) \sin(3\theta) d\theta \end{aligned}$$

$$\left[ \begin{array}{l} u = \cos(3\theta) \\ du = -3 \sin(3\theta) d\theta \\ -\frac{1}{3} du = \sin(3\theta) d\theta \end{array} \right]$$

$$\begin{aligned} &= -15 \int (1 - u^2) u^2 du \\ &= -15 \int (u^2 - u^4) du \\ &= 15 \int (u^4 - u^2) du \\ &= 15 \left( \frac{u^5}{5} - \frac{u^3}{3} \right) + C \\ &= 3u^5 - 5u^3 + C \\ &= 3 \cos^5(3\theta) - 5 \cos^3(3\theta) + C. \end{aligned}$$