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MATH 107 Quiz 1 (solutions)
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Evaluate the following integrals.

1. $\displaystyle\int_{-3}^{0} (p+2)^3\,dp$

   Solution. Let $u = p+2$. Then $du = dp$, so

$$\int_{-3}^{0} (p+2)^3\,dp = \int_{p=-3}^{p=0} u^3\,du = \left.\frac{u^4}{4}\right|_{p=-3}^{p=0} = \left.\frac{(p+2)^4}{4}\right|_{-3}^{0}$$
$$= \frac{(0+2)^4}{4} - \frac{(-3+2)^4}{4} = \frac{16}{4} - \frac{1}{4} = \frac{15}{4}.$$

2. $\displaystyle\int \frac{\sin(2t+1)}{\cos^2(2t+1)}\,dt$

   Solution. Let $u = \cos(2t+1)$. Then $du = -2\sin(2t+1)\,dt$, so $-\frac{1}{2}du = \sin(2t+1)\,dt$. So we have

$$\int \frac{\sin(2t+1)}{\cos^2(2t+1)}\,dt = -\frac{1}{2}\int \frac{1}{u^2}\,du = -\frac{1}{2}\int u^{-2}\,du = -\frac{1}{2}(-u^{-1}) + C$$
$$= -\frac{1}{2}\left(-\frac{1}{u}\right) + C = \frac{1}{2u} + C = \frac{1}{2\cos(2t+1)} + C.$$

   Alternatively, we can notice that

$$\int \frac{\sin(2t+1)}{\cos^2(2t+1)}\,dt = \int \sec(2t+1)\tan(2t+1)\,dt.$$

   Letting $u = \sec(2t+1)$, we find that $du = 2\sec(2t+1)\tan(2t+1)\,dt$, so $\frac{1}{2}du = \sec(2t+1)\tan(2t+1)\,dt$. Then

$$\int \sec(2t+1)\tan(2t+1)\,dt = \frac{1}{2}\int du = \frac{1}{2}u + C = \frac{1}{2}\sec(2t+1) + C.$$