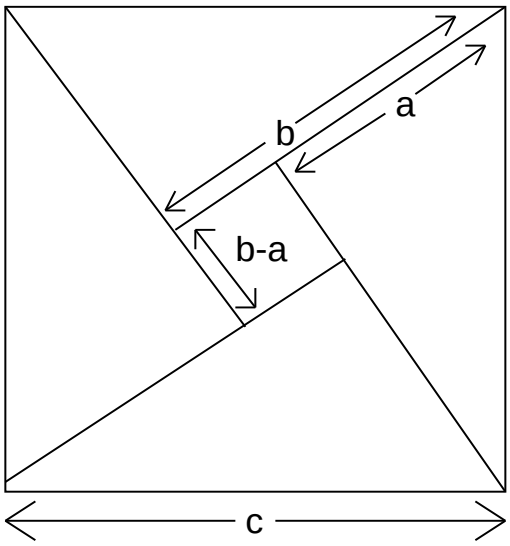
Fun with the right triangle.

The discovery of the theorem relating the legs and the hypotenuse of a right triangle is one of the most profound accomplishments in the history of mathematics. Simply stated, for any right triangle with legs a and b and hypotenuse c, $a^2 + b^2 = c^2$. The converse of the theorem states that if $a^2 + b^2 = c^2$ then the triangle is a right triangle. The Greek mathematician Pythagoras (572-497 B.C.) generally is credited with the discovery of the theorem. However some evidence suggests that the Chinese used a version of this formula in calculating measures for right triangles before Pythagoras, and Egyptian mathematicians may have used this right triangle relationship when building the pyramids.

An Elegant Proof

According to legend, the Hindu mathematician Bhaskara (1114-1185) drew this square and presented it to his peers with the simple statement "Behold!" Supposedly, he believed this construction presented a beautiful and obvious proof of the Pythagorean theorem. Use algebra to show why this is indeed a proof of the relationship $a^2 + b^2 = c^2$.



Right Triangles and Squares

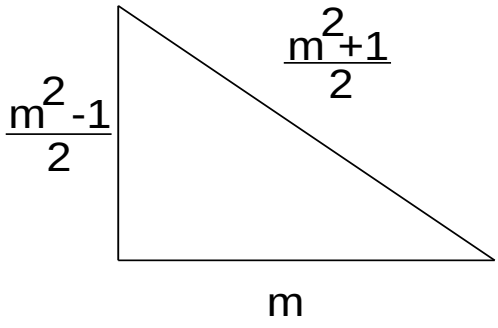
The 3-4-5 right triangle (meaning the sides have lengths 3, 4, and 5) was used to create the drawing for Bhaskara's picture. Do other right triangles also generate squares? Begin with the 5-12-13 triangle. Draw four of these triangles on graph paper, and cut them out. Can you arrange the four triangles in the same way that the 3-4-5 triangles were arranged to create a square? If so, what are the dimensions of this square? What is the shape in the center? Repeat this process with the 7-24-25 triangle. Choose any other right triangle, and see if you can form a square. Will this process work with any right triangle? Explain why or why not.

A Little Research

There are over 300 proofs of the right triangle relationship $a^2 + b^2 = c^2$. One was even discovered by a former president of the United States, James Garfield. Find a proof of the theorem other than the ones in this project, and explain how it works in your own words. A good place to start looking is on the web. Be sure to cite any resources you use in your report.

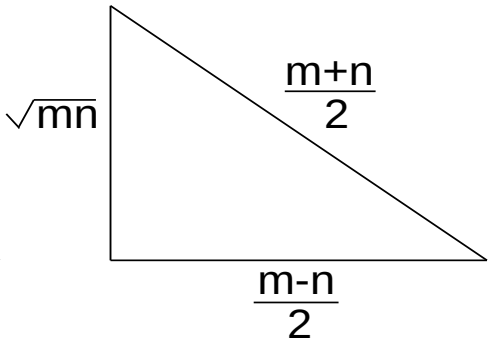
Generating Right Triangle Triples

A "Pythagorean triple" is a triple of integers (such as 3-4-5 and 5-12-13) satisfying the Pythagorean Theorem. Pythagorean triples can be generated using a variety of rules. Below are four different rules that have been used to find Pythagorean triples. For each of the rules, choose three values and show that the rule does indeed generate Pythagorean triples. Also, use algebra to prove that each rule satisfies the Pythagorean Theorem.



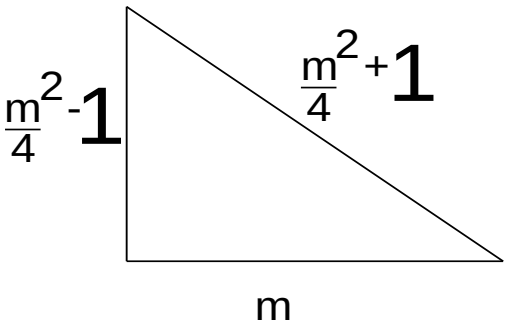
Rule of Pythagoras

m is any odd number



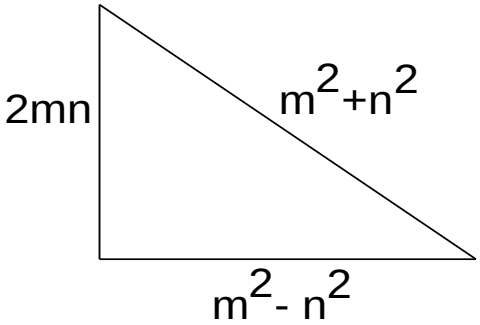
Rule of Euclid

m and n are both odd or both even, m>n, and the product mn must be a square number



Rule of Plato

m is any even number



Rule of Maseres

m and n can be any two integers and m>n