NAME:

MATH 103 Exam 4, version ((a))

25 November 2008

100 points

Instructions:

1. This exam has 7 pages (including this one and the formula sheet), which contain 7 questions and 2 bonus questions. Please check that you have all of the pages.
2. Answer all of the following questions clearly and completely. Justify all of your answers.
3. You may not use a book or any notes for this exam, except the formula sheet attached as the last page of the exam.
4. Give your answer to each problem completely and clearly in the space provided. You may use the back of the exam pages for scratch work; however, if you want this work to be considered, make note of it in the space provided for the problem.
5. Erase or cross out work you do not wish to be graded.
6. Credit, partial or full, will be given only if sufficient steps leading to the answers are shown.
7. You have 50 minutes to complete this exam.

Question 1. (10 points) If $\sin\theta = -\frac{63}{65}$ and θ is in Quadrant III, then what are $\cos\theta$ and $\tan\theta$?

Question 2. (10 points) Give an expression for a periodic function having an amplitude of 7, a period of 3π, and a y-intercept of 1.

Question 3. (20 points) Find the exact values of the following expressions.

(a) (5 points) $\sec \frac{7\pi}{6}$

(b) (5 points) $\sin^{-1}\left(-\frac{1}{2}\right)$

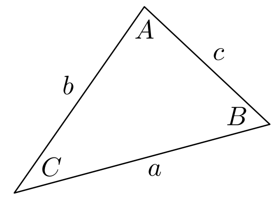
(c) (5 points) $\cos^{-1}\left(\cos \frac{13\pi}{7}\right)$

(d) (5 points) $\tan \frac{\pi}{8}$

Question 4. (15 points) Establish the identity: $\csc\theta - \sin\theta = \cos\theta\cot\theta$.

Question 5. (15 points) Establish the identity: $\dfrac{\cos(\alpha+\beta)}{\cos\alpha\cos\beta} = 1 - \tan\alpha\tan\beta$.

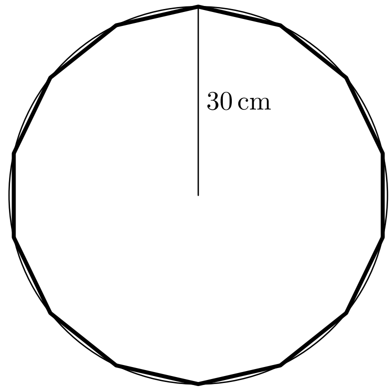
Question 6. (20 points) Solve the following triangles.



(a) (10 points) $a = 2,\ c = 1,\ C = 100°$

(b) (10 points) $a = 3,\ c = 2,\ B = 110°$

Question 7. (10 points) Find the area of a regular tetradecagon (14-sided polygon) inscribed in a circle of radius 30 cm, as shown below.



Bonus. (+4 points) Explain why

$$\cos 1^\circ + \cos 2^\circ + \cos 3^\circ + \cdots + \cos 358^\circ + \cos 359^\circ = -1.$$

Bonus. (+1 point) Find a common English word containing the letters KSG together and in that order.

NAME:

MATH 103 Exam 4, version ((b))

25 November 2008

100 points

Instructions:

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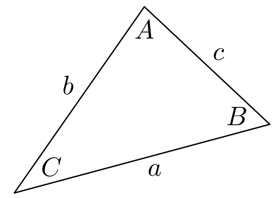
7. You have 50 minutes to complete this exam.

Question 1. (20 points) Solve the following triangles.



(a) (10 points) $a = 2,\ c = 1,\ C = 100°$

(b) (10 points) $a = 3,\ b = 4,\ C = 40°$

Question 2. (15 points) Establish the identity: $\sec\theta - \cos\theta = \sin\theta\tan\theta$.

Question 3. (15 points) Establish the identity: $\dfrac{\cos(\alpha+\beta)}{\cos\alpha\cos\beta} = 1 - \tan\alpha\tan\beta$.

Question 4. (20 points) Find the exact values of the following expressions.

(a) (5 points) $\cot \frac{5\pi}{3}$

(b) (5 points) $\cos^{-1}\left(-\frac{1}{2}\right)$

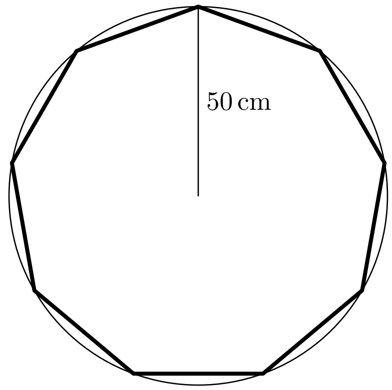
(c) (5 points) $\sin^{-1}\left(\sin \frac{3\pi}{5}\right)$

(d) (5 points) $\tan \frac{\pi}{12}$

Question 5. (10 points) If $\cos\theta = -\dfrac{55}{73}$ and θ is in Quadrant IV, then what are $\sin\theta$ and $\tan\theta$?

Question 6. (10 points) Give an expression for a periodic function having an amplitude of 4, a period of 5π, and a y-intercept of 0.

Question 7. (10 points) Find the area of a regular nonagon (9-sided polygon) inscribed in a circle of radius 50 cm, as shown below.



Bonus. (+4 points) Explain why

$$\cos 1^\circ + \cos 2^\circ + \cos 3^\circ + \cdots + \cos 358^\circ + \cos 359^\circ = -1.$$

Bonus. (+1 point) Find a common English word containing the letters KSG together and in that order.

Sum and difference formulas

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta}$

$\tan(\alpha-\beta) = \dfrac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta}$

Double-angle formulas

$\sin(2\theta) = 2\sin\theta\cos\theta$

$\cos(2\theta) = \cos^2\theta - \sin^2\theta$

$\cos(2\theta) = 1 - 2\sin^2\theta$

$\cos(2\theta) = 2\cos^2\theta - 1$

$\tan(2\theta) = \dfrac{2\tan\theta}{1-\tan^2\theta}$

Corollaries of double-angle formulas

$\sin^2\theta = \dfrac{1-\cos(2\theta)}{2}$

$\cos^2\theta = \dfrac{1+\cos(2\theta)}{2}$

$\tan^2\theta = \dfrac{1-\cos(2\theta)}{1+\cos(2\theta)}$

Half-angle formulas

$\sin\dfrac{\alpha}{2} = \pm\sqrt{\dfrac{1-\cos\alpha}{2}}$

$\cos\dfrac{\alpha}{2} = \pm\sqrt{\dfrac{1+\cos\alpha}{2}}$

$\tan\dfrac{\alpha}{2} = \pm\sqrt{\dfrac{1-\cos\alpha}{1+\cos\alpha}}$

$\tan\dfrac{\alpha}{2} = \dfrac{1-\cos\alpha}{\sin\alpha} = \dfrac{\sin\alpha}{1+\cos\alpha}$

Product-to-sum formulas

$\sin\alpha\sin\beta = \dfrac{1}{2}\big[\cos(\alpha-\beta) - \cos(\alpha+\beta)\big]$

$\cos\alpha\cos\beta = \dfrac{1}{2}\big[\cos(\alpha-\beta) + \cos(\alpha+\beta)\big]$

$\sin\alpha\cos\beta = \dfrac{1}{2}\big[\sin(\alpha+\beta) + \sin(\alpha-\beta)\big]$

Sum-to-product formulas

$\sin\alpha + \sin\beta = 2\sin\dfrac{\alpha+\beta}{2}\cos\dfrac{\alpha-\beta}{2}$

$\sin\alpha - \sin\beta = 2\sin\dfrac{\alpha-\beta}{2}\cos\dfrac{\alpha+\beta}{2}$

$\cos\alpha + \cos\beta = 2\cos\dfrac{\alpha+\beta}{2}\cos\dfrac{\alpha-\beta}{2}$

$\cos\alpha - \cos\beta = -2\sin\dfrac{\alpha+\beta}{2}\sin\dfrac{\alpha-\beta}{2}$

Law of Sines

$\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

Law of Cosines

$c^2 = a^2 + b^2 - 2ab\cos C$

$b^2 = a^2 + c^2 - 2ac\cos B$

$a^2 = b^2 + c^2 - 2bc\cos A$

Area of a triangle

In the following formulas, K denotes the area of a triangle.

$K = \dfrac{1}{2}bh$

$K = \dfrac{1}{2}ab\sin C$

$K = \dfrac{1}{2}bc\sin A$

$K = \dfrac{1}{2}ac\sin B$

$K = \sqrt{s(s-a)(s-b)(s-c)}$,

where $s = \dfrac{a+b+c}{2}$