Math 103: College Algebra and Trigonometry
Exam 4 review problems

Problem 1. Find the exact value of each of the following expressions. (Do not use a calculator.)

(a) $\cos 390°$ (b) $\cot \frac{7\pi}{6}$ (c) $\csc 450°$ (d) $\sin \frac{33\pi}{4}$ (e) $\tan 225°$

(f) $\sin\left(\frac{-\pi}{6}\right)$ (g) $\sec 195°$ (h) $\tan(-6\pi) + \cos\frac{9\pi}{4}$ (i) $\cos\frac{5\pi}{12}$

(j) $\sin^{-1}\frac{\sqrt{2}}{2}$ (k) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ (l) $\tan^{-1}\sqrt{3}$ (m) $\sec\left[\sec^{-1}(7)\right]$

(n) $\cos^{-1}\left(\cos\frac{4\pi}{3}\right)$ (o) $\sec\left(\cos^{-1}\frac{1}{2}\right)$ (p) $\cot\left[\cos^{-1}\left(-\frac{\sqrt{3}}{3}\right)\right]$

(q) $\cos 13° \cos 32° - \sin 13° \sin 32°$ (r) $\cos\left(-\frac{\pi}{8}\right)$

Problem 2. If $\sin\theta < 0$ and $\sec\theta > 0$, then which quadrant is θ in?

Problem 3. Suppose θ is an angle in quadrant II and $\sin\theta = \frac{65}{97}$. Find the exact value of each of the remaining trigonometric functions of θ.

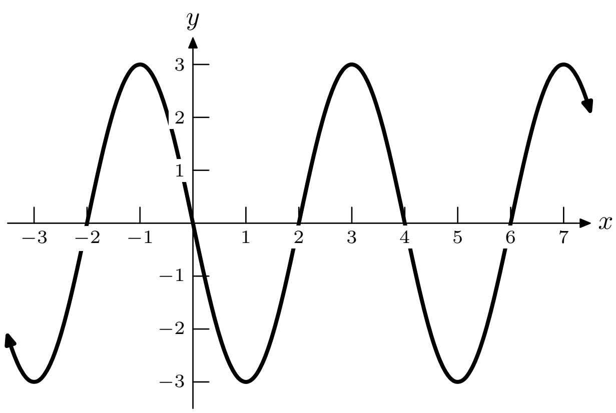
Problem 4. For each of the six trigonometric functions listed below, give the domain and range of the function, and state whether the function is even, odd, or neither.

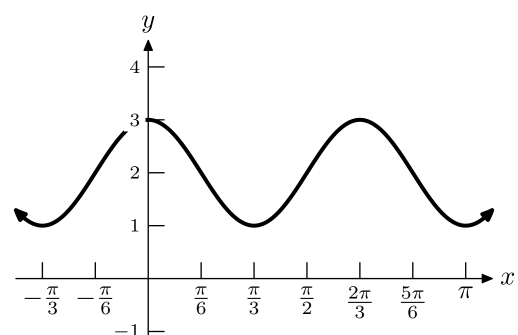
$$\sin\theta, \quad \cos\theta, \quad \tan\theta, \quad \cot\theta, \quad \sec\theta, \quad \csc\theta.$$

Problem 5. What are the domain and range of each of the six inverse trigonometric functions: $\sin^{-1}\theta$, $\cos^{-1}\theta$, $\tan^{-1}\theta$, $\cot^{-1}\theta$, $\sec^{-1}\theta$, and $\csc^{-1}\theta$?

Problem 6. Determine the amplitude and period of $y = -8\cos\left(\frac{5}{3}x\right)$ without graphing.

Problem 7. Find an equation for each graph below.





Problem 8. What is the y-intercept of the graph of each of the following trigonometric functions: $y = \sin x$, $y = \cos x$, $y = \tan x$, $y = \cot x$, $y = \sec x$, and $y = \csc x$?

Problem 9. Write $\dfrac{\cos\theta}{1-\sin\theta}$ as the sum of two trigonometric functions of θ.

[Hint: Start by multiplying it by $\dfrac{1+\sin\theta}{1+\sin\theta}$. Note that you are really just multiplying it by 1, so you are not changing the *value* of the fraction.]

Problem 10. Factor and simplify: $\dfrac{\cos^2\theta - 1}{\cos^2\theta - \cos\theta}$.

Problem 11. Establish the following identities.

(a) $(\cos\theta)(\tan\theta + \cot\theta) = \csc\theta$

(b) $\dfrac{\cos(\alpha+\beta)}{\cos\alpha\cos\beta} = 1 - \tan\alpha\tan\beta$

(c) $9\sec^2\theta - 5\tan^2\theta = 5 + 4\sec^2\theta$

(d) $\sec^2 u - (\sin^2 u)(\sec^2 u + 2) = \cos(2u)$

(e) $\dfrac{\tan\theta - \cot\theta}{\tan\theta + \cot\theta} = \sin^2\theta - \cos^2\theta$

(f) $\cot(2\theta) = \dfrac{\cot^2\theta - 1}{2\cot\theta}$

(g) $\dfrac{\cos\theta + \cos(3\theta)}{2\cos(2\theta)} = \cos\theta$

Problem 12. Solve each of the following equations on the interval $0 \le \theta < 2\pi$.

(a) $\sqrt{3}\cot\theta + 1 = 0$

(b) $3\cos(5\theta) - 2 = 1$

(c) $\sin^2\theta - \cos^2\theta = 1 + \cos\theta$

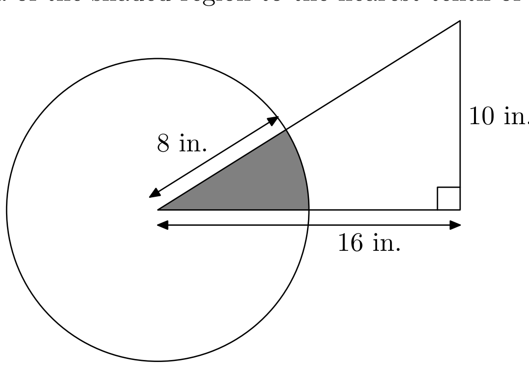
(d) $\csc^2\theta = \cot\theta + 1$

(e) $\sin\theta - \cos\theta = 0$

Problem 13. One angle of a right triangle measures 20°, and the hypotenuse has a length of 9. Find the measure of the other acute angle and the lengths of the two legs.

Problem 14. Wichita, Kansas is 215 km directly southwest of Topeka, Kansas. Grand Island, Nebraska is 310 km directly northwest of Topeka. Ignoring the curvature of the Earth's surface, how far is Wichita from Grand Island, as the crow flies? In what direction (bearing) must this crow fly in order to go directly from Grand Island to Wichita?

Problem 15. In the figure below, the center of the circle is one of the vertices of the triangle. Find the area of the shaded region to the nearest tenth of a square inch.



Problem 16. George Washington Gale Ferris, Jr. designed the original Ferris wheel for the 1893 World's Columbian Exposition in Chicago, Illinois. The wheel had 36 equally spaced cars each the size of a school bus. The distance between adjacent cars was approximately 22 feet. Determine the diameter of the wheel to the nearest foot.

Problem 17. Solve each triangle. [It may be possible that two solutions exist, or that no solutions exist.]

(a) $A = 50°,\ C = 20°,\ a = 3$

(b) $a = 2,\ c = 1,\ C = 100°$

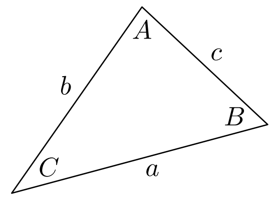
(c) $a = 2,\ b = 2,\ C = 50°$

(d) $b = 5,\ c = 3,\ B = 100°$

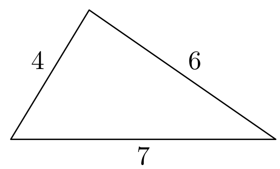
(e) $b = 2,\ c = 3,\ B = 40°$

(f) $a = 4,\ b = 3,\ c = 6$

(g) $a = 33,\ b = 56,\ c = 65$



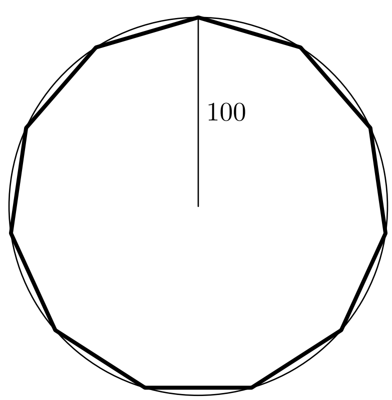
Problem 18. Find the area of the triangle below.



Problem 19. A yardstick and a meterstick are laid on the ground, touching at one end, to form an angle of 40°. A line is drawn from the other end of the yardstick to the other end of the meterstick, making a triangle. Find the area of this triangle. [Be sure to include units in your answer!]

Problem 20. Two points are drawn on a circle of radius 5 cubits. The distance between the two points, measured along the straight line connecting them, is 3 cubits. Find the area of the triangle formed by the two points and the center of the circle.

Problem 21. Find the area of a regular hendecagon (11-sided polygon) inscribed in a circle of radius 100, as shown below.



Bonus. Find the exact value of $\cos 1° + \cos 2° + \cos 3° + \cdots + \cos 358° + \cos 359°$.

Sum and difference formulas

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\tan(\alpha+\beta) = \dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

$\tan(\alpha-\beta) = \dfrac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$

Double-angle formulas

$\sin(2\theta) = 2\sin\theta\cos\theta$

$\cos(2\theta) = \cos^2\theta - \sin^2\theta$

$\cos(2\theta) = 1 - 2\sin^2\theta$

$\cos(2\theta) = 2\cos^2\theta - 1$

$\tan(2\theta) = \dfrac{2\tan\theta}{1-\tan^2\theta}$

Corollaries of double-angle formulas

$\sin^2\theta = \dfrac{1-\cos(2\theta)}{2}$

$\cos^2\theta = \dfrac{1+\cos(2\theta)}{2}$

$\tan^2\theta = \dfrac{1-\cos(2\theta)}{1+\cos(2\theta)}$

Half-angle formulas

$\sin\dfrac{\alpha}{2} = \pm\sqrt{\dfrac{1-\cos\alpha}{2}}$

$\cos\dfrac{\alpha}{2} = \pm\sqrt{\dfrac{1+\cos\alpha}{2}}$

$\tan\dfrac{\alpha}{2} = \pm\sqrt{\dfrac{1-\cos\alpha}{1+\cos\alpha}}$

$\tan\dfrac{\alpha}{2} = \dfrac{1-\cos\alpha}{\sin\alpha} = \dfrac{\sin\alpha}{1+\cos\alpha}$

Product-to-sum formulas

$\sin\alpha\sin\beta = \dfrac{1}{2}\left[\cos(\alpha-\beta) - \cos(\alpha+\beta)\right]$

$\cos\alpha\cos\beta = \dfrac{1}{2}\left[\cos(\alpha-\beta) + \cos(\alpha+\beta)\right]$

$\sin\alpha\cos\beta = \dfrac{1}{2}\left[\sin(\alpha+\beta) + \sin(\alpha-\beta)\right]$

Sum-to-product formulas

$\sin\alpha + \sin\beta = 2\sin\dfrac{\alpha+\beta}{2}\cos\dfrac{\alpha-\beta}{2}$

$\sin\alpha - \sin\beta = 2\sin\dfrac{\alpha-\beta}{2}\cos\dfrac{\alpha+\beta}{2}$

$\cos\alpha + \cos\beta = 2\cos\dfrac{\alpha+\beta}{2}\cos\dfrac{\alpha-\beta}{2}$

$\cos\alpha - \cos\beta = -2\sin\dfrac{\alpha+\beta}{2}\sin\dfrac{\alpha-\beta}{2}$

Law of Sines

$\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

Law of Cosines

$c^2 = a^2 + b^2 - 2ab\cos C$

$b^2 = a^2 + c^2 - 2ac\cos B$

$a^2 = b^2 + c^2 - 2bc\cos A$

Area of a triangle

In the following formulas, K denotes the area of a triangle.

$K = \dfrac{1}{2}bh$

$K = \dfrac{1}{2}ab\sin C$

$K = \dfrac{1}{2}bc\sin A$

$K = \dfrac{1}{2}ac\sin B$

$K = \sqrt{s(s-a)(s-b)(s-c)}$,

where $s = \dfrac{a+b+c}{2}$