Math 103: College Algebra and Trigonometry

Exam 4 review answers

Answer 1. Several of these answers may be written in different forms. Two equivalent forms of the answer are given below where appropriate. Other forms of these answers may be possible.

(a) $\dfrac{\sqrt{3}}{2}$ (b) $\sqrt{3}$ (c) 1 (d) 1

(f) $-\dfrac{1}{2}$ (g) $-\sqrt{\dfrac{2}{1+\frac{\sqrt{3}}{2}}} = -\dfrac{4}{\sqrt{2}+\sqrt{6}}$ (h) $\dfrac{\sqrt{2}}{2} = \dfrac{1}{\sqrt{2}}$ (i) $\dfrac{\sqrt{2-\sqrt{3}}}{2} = \dfrac{\sqrt{6}-\sqrt{2}}{4}$

(j) $\dfrac{\pi}{4} = 45°$ (k) $\dfrac{5\pi}{6} = 150°$ (l) $\dfrac{\pi}{3} = 60°$ (m) 7

(n) $\dfrac{2\pi}{3} = 120°$ (o) 2 (p) $-\dfrac{\sqrt{2}}{2} = -\dfrac{1}{\sqrt{2}}$

(q) $\dfrac{\sqrt{2}}{2} = \dfrac{1}{\sqrt{2}}$ (r) $\dfrac{\sqrt{2+\sqrt{2}}}{2}$

Answer 2. Quadrant IV

Answer 3. $\sin\theta = \dfrac{65}{97}$; $\cos\theta = -\dfrac{72}{97}$; $\tan\theta = -\dfrac{65}{72}$; $\cot\theta = -\dfrac{72}{65}$; $\sec\theta = -\dfrac{97}{72}$; $\csc\theta = \dfrac{97}{65}$

Answer 4.

Function	Domain	Range	Even/odd
$\sin\theta$	$(-\infty, \infty)$	$[-1, 1]$	Odd
$\cos\theta$	$(-\infty, \infty)$	$[-1, 1]$	Even
$\tan\theta$	All real numbers except $\pi/2 + k\pi$ for integers k	$(-\infty, \infty)$	Odd
$\cot\theta$	All real numbers except $\pi/2 + k\pi$ for integers k	$(-\infty, \infty)$	Odd
$\sec\theta$	All real numbers except $\pi/2 + k\pi$ for integers k	$(-\infty, -1] \cup [1, \infty)$	Even
$\csc\theta$	All real numbers except $k\pi$ for integers k	$(-\infty, -1] \cup [1, \infty)$	Odd

Answer 5. While not strictly an error, there was a typo in the problem on the question sheet. Writing the inverse trigonometric functions as $\sin^{-1}\theta$, $\cos^{-1}\theta$, etc. is misleading, since θ usually represents an angle, but the arguments to the inverse trigonometric functions are not angles. (The *outputs* of the inverse trigonometric functions are angles, not the *inputs*.) It would have been better to have written $\sin^{-1} x$, $\cos^{-1} x$, etc.

Function	Domain	Range
$\sin^{-1} x$	$[-1, 1]$	$[-\pi/2, \pi/2]$
$\cos^{-1} x$	$[-1, 1]$	$[0, \pi]$
$\tan^{-1} x$	$(-\infty, \infty)$	$(\pi/2, \pi/2)$
$\cot^{-1} x$	$(-\infty, \infty)$	$(\pi/2, \pi/2)$
$\sec^{-1} x$	$(-\infty, -1] \cup [1, \infty)$	$[0, \pi/2) \cup (\pi/2, \pi]$
$\csc^{-1} x$	$(-\infty, -1] \cup [1, \infty)$	$[-\pi/2, 0) \cup (0, \pi/2]$

Answer 6. The amplitude is 8, and the period is $\dfrac{6\pi}{5}$.

Answer 7. An equation for the graph on the left is $y = -3\sin\left(\frac{\pi x}{2}\right)$. An equation for the graph on the right is $y = \cos(3x) + 2$.

Answer 8.

Function	y-intercept
$\sin x$	0
$\cos x$	1
$\tan x$	0
$\cot x$	no y-intercept ($\cot x$ is undefined at $x = 0$)
$\sec x$	1
$\csc x$	no y-intercept ($\csc x$ is undefined at $x = 0$)

Answer 9. $\dfrac{\cos\theta}{1-\sin\theta} = \sec\theta + \tan\theta$

Answer 10. $\dfrac{\cos^2\theta - 1}{\cos^2\theta - \cos\theta} = 1 + \sec\theta$

Answer 11. Other methods are possible.

(a)
$$\begin{aligned}(\cos\theta)(\tan\theta + \cot\theta) &= (\cos\theta)\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)\\ &= (\cos\theta)\left(\frac{\sin^2\theta}{\sin\theta\cos\theta} + \frac{\cos^2\theta}{\sin\theta\cos\theta}\right)\\ &= (\cos\theta)\left(\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}\right)\\ &= (\cos\theta)\left(\frac{1}{\sin\theta\cos\theta}\right)\\ &= \frac{\cos\theta}{\sin\theta\cos\theta}\\ &= \frac{1}{\sin\theta}\\ &= \csc\theta.\end{aligned}$$

(b)
$$\begin{aligned}\frac{\cos(\alpha+\beta)}{\cos\alpha\cos\beta} &= \frac{\cos\alpha\cos\beta - \sin\alpha\sin\beta}{\cos\alpha\cos\beta}\\ &= \frac{\cos\alpha\cos\beta}{\cos\alpha\cos\beta} - \frac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}\\ &= 1 - \left(\frac{\sin\alpha}{\cos\alpha}\right)\left(\frac{\sin\beta}{\cos\beta}\right)\\ &= 1 - \tan\alpha\tan\beta.\end{aligned}$$

(c)
$$\begin{aligned}9\sec^2\theta - 5\tan^2\theta &= 4\sec^2\theta + 5\sec^2\theta - 5\tan^2\theta\\ &= 4\sec^2\theta + 5(\sec^2\theta - \tan^2\theta)\\ &= 4\sec^2\theta + 5(1)\\ &= 5 + 4\sec^2\theta.\end{aligned}$$

(d) $\sec^2 u - (\sin^2 u)(\sec^2 u + 2) = \sec^2 u - \sin^2 u \sec^2 u - 2\sin^2 u$
$= (\sec^2 u)(1 - \sin^2 u) - 2\sin^2 u$
$= \left(\frac{1}{\cos^2 u}\right)(\cos^2 u) - 2\sin^2 u$
$= \left(\frac{\cos^2 u}{\cos^2 u}\right) - 2\sin^2 u$
$= 1 - 2\sin^2 u$
$= \cos(2u).$

(e) $\frac{\tan\theta - \cot\theta}{\tan\theta + \cot\theta} = \frac{\left(\frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\sin\theta}\right)}{\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)}$
$= \frac{\left(\frac{\sin^2\theta}{\sin\theta\cos\theta} - \frac{\cos^2\theta}{\sin\theta\cos\theta}\right)}{\left(\frac{\sin^2\theta}{\sin\theta\cos\theta} + \frac{\cos^2\theta}{\sin\theta\cos\theta}\right)}$
$= \frac{\left(\frac{\sin^2\theta - \cos^2\theta}{\sin\theta\cos\theta}\right)}{\left(\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}\right)}$
$= \frac{\left(\frac{\sin^2\theta - \cos^2\theta}{\sin\theta\cos\theta}\right)}{\left(\frac{1}{\sin\theta\cos\theta}\right)}$
$= \left(\frac{\sin^2\theta - \cos^2\theta}{\sin\theta\cos\theta}\right)\left(\frac{\sin\theta\cos\theta}{1}\right)$
$= \sin^2\theta - \cos^2\theta.$

(f) $\frac{\cot^2\theta - 1}{2\cot\theta} = \frac{\left(\frac{\cos^2\theta}{\sin^2\theta} - 1\right)}{2\left(\frac{\cos\theta}{\sin\theta}\right)}$
$= \frac{\left(\frac{\cos^2\theta}{\sin^2\theta} - \frac{\sin^2\theta}{\sin^2\theta}\right)}{\left(\frac{2\cos\theta}{\sin\theta}\right)}$
$= \frac{\left(\frac{\cos^2\theta - \sin^2\theta}{\sin^2\theta}\right)}{\left(\frac{2\cos\theta}{\sin\theta}\right)}$
$= \left(\frac{\cos^2\theta - \sin^2\theta}{\sin^2\theta}\right)\left(\frac{\sin\theta}{2\cos\theta}\right)$
$= \frac{(\cos^2\theta - \sin^2\theta)(\sin\theta)}{2\sin^2\theta\cos\theta}$
$= \frac{\cos^2\theta - \sin^2\theta}{2\sin\theta\cos\theta}$
$= \frac{\cos(2\theta)}{\sin(2\theta)}$
$= \cot(2\theta).$

(g) $\frac{\cos\theta + \cos(3\theta)}{2\cos(2\theta)} = \frac{2\cos\left(\frac{\theta+3\theta}{2}\right)\cos\left(\frac{\theta-3\theta}{2}\right)}{2\cos(2\theta)}$
$= \frac{2\cos\left(\frac{4\theta}{2}\right)\cos\left(\frac{-2\theta}{2}\right)}{2\cos(2\theta)}$
$= \frac{2\cos(2\theta)\cos(-\theta)}{2\cos(2\theta)}$
$= \cos(-\theta)$
$= \cos\theta.$

Answer 12.

(a) Solution set: $\left\{\frac{2\pi}{3}, \frac{5\pi}{3}\right\}$

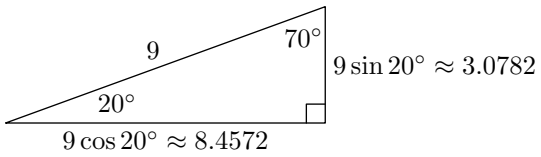
(b) Solution set: $\left\{0, \frac{2\pi}{5}, \frac{4\pi}{5}, \frac{6\pi}{5}, \frac{8\pi}{5}\right\}$

(c) Solution set: $\left\{\frac{\pi}{2}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{3\pi}{2}\right\}$

(d) Solution set: $\left\{\frac{\pi}{4}, \frac{\pi}{2}, \frac{5\pi}{4}, \frac{3\pi}{2}\right\}$

(e) Solution set: $\left\{\frac{\pi}{4}, \frac{5\pi}{4}\right\}$

Answer 13.



Answer 14. Wichita is approximately 377 km from Grand Island. A crow flying directly from Grand Island to Wichita must fly on a bearing of about S10.26°E, that is, 10.26° east of south.

Answer 15. To the nearest tenth of a square inch, the area of the shaded region is 17.9 in^2.

Answer 16. To the nearest foot, the diameter of the wheel was 252 feet.

Answer 17.

(a) $B = 110°,\ b \approx 3.6800,\ c \approx 1.3394$

(b) No solution

(c) $c \approx 1.6905,\ A = 65°,\ B = 65°$

(d) $a \approx 3.5128,\ A \approx 43.78°,\ C \approx 36.22°$

(e) Two solutions:

$$C \approx 74.62°,\ A \approx 65.38°,\ a \approx 2.8286 \quad \text{or} \quad C \approx 105.38°,\ A \approx 34.62°,\ a \approx 1.7676$$

(f) $A \approx 36.34°,\ B \approx 26.38°,\ C \approx 117.28°$

(g) $A \approx 30.51°,\ B \approx 59.49°,\ C = 90°$

Answer 18. The area of the triangle is $\dfrac{3\sqrt{255}}{4} \approx 11.9765$.

Answer 19. The area of the triangle is approximately 0.2939 m^2, or approximately 2939 cm^2, or approximately 455.52 in^2, or approximately 3.1633 ft^2.

Answer 20. The area of the triangle is $\dfrac{3\sqrt{91}}{4}$ square cubits, or approximately 7.1545 square cubits.

Answer 21. The area of the hendecagon is approximately 29 735. [Note that the area of the circle is approximately 31 416, so our answer for the area of the hendecagon makes sense, since it covers nearly all of the circle.]

Bonus. -1