

NAME:

Math 103 Exam 4 retake

12 December 2008

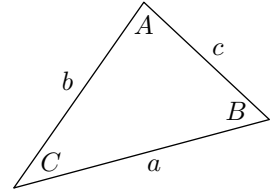
100 points

Instructions:

1. This exam has 7 pages (including this one and the formula sheet), which contain 7 questions and 2 bonus questions. Please check that you have all of the pages.
 2. Answer all of the following questions clearly and completely. Justify all of your answers.
 3. You may not use a book or any notes for this exam, except the formula sheet attached as the last page of the exam.
 4. Give your answer to each problem completely and clearly in the space provided. You may use the back of the exam pages for scratch work; however, if you want this work to be considered, make note of it in the space provided for the problem.
 5. Erase or cross out work you do not wish to be graded.
 6. Credit, partial or full, will be given only if sufficient steps leading to the answers are shown.
 7. You have 50 minutes to complete this exam.
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Question 1. (20 points) Solve the following triangles.

(a) (10 points) $b = 4$, $c = 5$, $B = 95^\circ$



(b) (10 points) $a = 2$, $b = 2$, $C = 50^\circ$

Question 2. (20 points) Find the exact values of the following expressions.

(a) (5 points) $\csc \frac{7\pi}{4}$

(b) (5 points) $\cos^{-1} \left(-\frac{\sqrt{3}}{2} \right)$

(c) (5 points) $\tan^{-1} \left(\tan \frac{16\pi}{9} \right)$

(d) (5 points) $\tan \frac{3\pi}{8}$

Question 3. (20 points) Consider the equation $4 \sin^2 \theta + 1 = 4$.

(a) (10 points) Find the *general* solution to this equation (with no restriction on the value of θ).

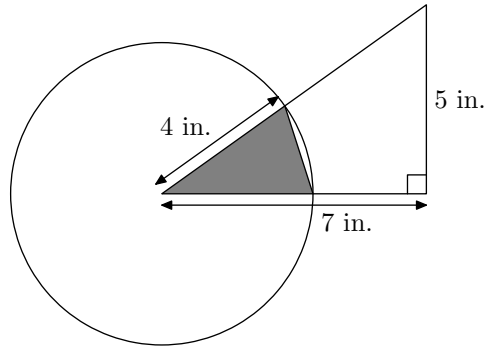
(b) (10 points) Find the solutions to this equation on the interval $0 \leq \theta < 2\pi$.

Question 4. (10 points) Establish the identity: $\csc(2\theta) = \frac{1}{2} \sec \theta \csc \theta$.

Question 5. (10 points) If $\tan \theta = -\frac{18}{80}$ and θ is in Quadrant II, then what are $\sin \theta$ and $\cos \theta$?

Question 6. (10 points) Let $f(x) = -4\sin(5\pi x)$. What are the amplitude and the period of f ?

Question 7. (10 points) In the figure below, the center of the circle is one of the corners of the triangle. Find the area of the shaded triangle.



Bonus. (+3 points) Explain why

$$\cos 1^\circ + \cos 2^\circ + \cos 3^\circ + \cdots + \cos 358^\circ + \cos 359^\circ = -1.$$

Bonus. (+1 point) Find a common English word containing the letters KSG together and in that order.

Sum and difference formulas

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

Double-angle formulas

$$\sin(2\theta) = 2 \sin \theta \cos \theta$$

$$\cos(2\theta) = \cos^2 \theta - \sin^2 \theta$$

$$\cos(2\theta) = 1 - 2 \sin^2 \theta$$

$$\cos(2\theta) = 2 \cos^2 \theta - 1$$

$$\tan(2\theta) = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

Corollaries of double-angle formulas

$$\sin^2 \theta = \frac{1 - \cos(2\theta)}{2}$$

$$\cos^2 \theta = \frac{1 + \cos(2\theta)}{2}$$

$$\tan^2 \theta = \frac{1 - \cos(2\theta)}{1 + \cos(2\theta)}$$

Half-angle formulas

$$\sin \frac{\alpha}{2} = \pm \sqrt{\frac{1 - \cos \alpha}{2}}$$

$$\cos \frac{\alpha}{2} = \pm \sqrt{\frac{1 + \cos \alpha}{2}}$$

$$\tan \frac{\alpha}{2} = \pm \sqrt{\frac{1 - \cos \alpha}{1 + \cos \alpha}}$$

$$\tan \frac{\alpha}{2} = \frac{1 - \cos \alpha}{\sin \alpha} = \frac{\sin \alpha}{1 + \cos \alpha}$$

Product-to-sum formulas

$$\sin \alpha \sin \beta = \frac{1}{2} [\cos(\alpha - \beta) - \cos(\alpha + \beta)]$$

$$\cos \alpha \cos \beta = \frac{1}{2} [\cos(\alpha - \beta) + \cos(\alpha + \beta)]$$

$$\sin \alpha \cos \beta = \frac{1}{2} [\sin(\alpha + \beta) + \sin(\alpha - \beta)]$$

Sum-to-product formulas

$$\sin \alpha + \sin \beta = 2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$\sin \alpha - \sin \beta = 2 \sin \frac{\alpha - \beta}{2} \cos \frac{\alpha + \beta}{2}$$

$$\cos \alpha + \cos \beta = 2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$\cos \alpha - \cos \beta = -2 \sin \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2}$$

Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Law of Cosines

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Area of a triangle

In the following formulas, K denotes the area of a triangle.

$$K = \frac{1}{2}bh$$

$$K = \frac{1}{2}ab \sin C$$

$$K = \frac{1}{2}bc \sin A$$

$$K = \frac{1}{2}ac \sin B$$

$$K = \sqrt{s(s-a)(s-b)(s-c)},$$

$$\text{where } s = \frac{a+b+c}{2}$$