Math 103: College Algebra and Trigonometry

Exam 3 review problems

Problem 1. The function $f(x) = x^3 + 5$ is one-to-one. Find its inverse function and check your answer.

Problem 2.

(a) Graph the function $f(x) = 3^x$.
(b) Use your graph from part (a) to graph the function $g(x) = 3^{-x}$.
(c) Use your graph from part (b) to graph the function $h(x) = 3^{-x} - 2$.

Problem 3. Solve $3^{x^2} \cdot 27^x = 9^9$.

Problem 4. What is the domain of the function $f(x) = \ln x$?

Problem 5. What is the domain of the function $f(x) = \log_{281}(x^2 + 4x - 5)$? [Hint: You will need to solve a polynomial inequality.]

Problem 6. Find the exact value of $3\log_{12} 2 + \log_{12} 18$ without using a calculator. [You can use a calculator to check your answer, of course.]

Problem 7. Solve $\log_2(x+1) + \log_2(x+3) = 3$. Check your answer. [Remember to make sure your solutions are in the domain of both $\log_2(x+1)$ and $\log_2(x+3)$ so that they really are solutions to the original equation. You might have to discard solutions that aren't in the domain.]

Problem 8. You invest \$1000 at 5% compounded semiannually (twice a year). How long will it take until you have at least \$1300?

Problem 9. How much money should you invest now in order to get \$500 after 5 years at 3% compounded continuously?

Problem 10. A dose of 13 mg of morphine is injected into a patient's bloodstream. The amount of morphine in the patient's bloodstream follows an exponential decay model with a half-life of 2.9 hours. How much morphine is in the patient's bloodstream after 1 hour? After how much time has the amount of morphine in the patient's bloodstream fallen to 10% of the initial dose?

Problem 11. A certain quantity of a radioactive substance is produced in a nuclear fission reaction. The quantity of this substance decays over time, following an exponential decay model. It is found that there are 78.25 grams of this substance remaining 2 hours after the reaction, and 4 hours after the reaction there are 72.03 grams left.

(a) Find the value of k in the exponential decay model for this substance. [Hint: It is easiest to let $t = 0$ correspond to the point in time 2 hours after the reaction, so that you know the value of A_0. If you do this, remember that the point in time 4 hours after the reaction will not be $t = 4$!]
(b) How much of this substance was originally produced in the reaction? [If in part (a) you used $t = 0$ to represent the point in time 2 hours *after* the reaction, then the first thing you must determine is what value of t corresponds to the reaction itself.]
(c) What is the half-life of this substance? [This substance in this problem is supposed to be zirconium-97, so you can check your answer by searching for the half-life of zirconium-97 on the Internet. The Wikipedia article "Isotopes of zirconium" is helpful.]

Problem 12.

(a) Convert an angle of $225°$ to radians. Express your answer as a multiple of π (in other words, do not approximate your answer with a decimal).

(b) Convert an angle of $-\dfrac{7\pi}{12}$ radians to degrees.

Problem 13. Suppose θ is an acute angle and $\tan\theta = \dfrac{15}{8}$. Find the values of the other five trigonometric functions of θ.

Problem 14. Find the exact value of $(\sec^3 57°)(\sin 33°) - \tan^2 57°$ without using a calculator. [Hint: Use the fundamental identities and the complementary angle theorem.]

Problem 15. Find the exact value of $\sin\dfrac{\pi}{6} + \cos^2\dfrac{\pi}{4}$ without using a calculator.

Problem 16. There is a flat field with a single large oak tree in the middle. Directly east of the tree, some distance away, is a white fencepost. A surveyor is standing 360 yards directly south of the tree. She measures the angle between her line of sight to the tree and her line of sight to the fencepost to be $42.1°$. To the nearest yard, how far is the fencepost from the tree? How far is the surveyor from the fencepost?

Here is some graph paper for you to use for something if you like.