

**NAME:**

Math 103 Exam 3 retake

12 December 2008

100 points

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**Instructions:**

1. This exam has 7 pages (including this one), which contain 8 problems and one bonus problem. Please check that you have all of the pages.
  2. Answer all of the following questions clearly and completely. Justify all of your answers.
  3. You may not use a book or any notes for this exam.
  4. Give your answer to each problem completely and clearly in the space provided. You may use the back of the exam pages for scratch work; however, if you want this work to be considered, make note of it in the space provided for the problem.
  5. Erase or cross out work you do not wish to be graded.
  6. Credit, partial or full, will be given only if sufficient steps leading to the answers are shown.
  7. You have 50 minutes to complete this exam.
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**Problem 1.** (10 points) Be sure to show work for both parts of this problem, even if you can do them in your head, so that I can see that you understand how to do these conversions and that you aren't just using your calculator.

(a) (5 points) Convert  $\frac{4\pi}{3}$  radians to degrees.

(b) (5 points) Convert  $165^\circ$  to radians. Express your answer as a multiple of  $\pi$  (in other words, do not approximate your answer with a decimal).

**Problem 2.** (12 points) Find the exact value of  $2 \log_3 45 - \log_3 25$  without using a calculator.

**Problem 3.** (12 points) Suppose  $\theta$  is an acute angle and  $\cos \theta = \frac{28}{53}$ . Find the values of the other five trigonometric functions of  $\theta$ .

**Problem 4.** (14 points) The function  $f(x) = \frac{3}{x-10}$  is one-to-one. Find its inverse function  $f^{-1}$  and check your answer.

**Problem 5.** (14 points) A horizontal awning 10 feet wide is to be attached to the side of a building. A supporting chain will run from the outer edge of the awning to the side of the building above the point the awning is attached. The angle between the awning and the chain must be at least  $37^\circ$  in order to support the weight of the awning. The builder wants to use the least possible amount of chain.

(a) (7 points) How long should the chain be?

(b) (7 points) How far above the awning should the chain be attached to the building?

**Problem 6.** (16 points) Remember that radioactive decay follows an exponential decay model, so the amount  $A(t)$  of a radioactive substance at time  $t$  is given by

$$A(t) = A_0 e^{kt},$$

where  $A_0$  is the amount of the substance at time  $t = 0$ . A sample of 100 grams of radioactive rhenium-182 is produced in a reaction. After 22 hours, there are 78.80 grams of rhenium-182 remaining.

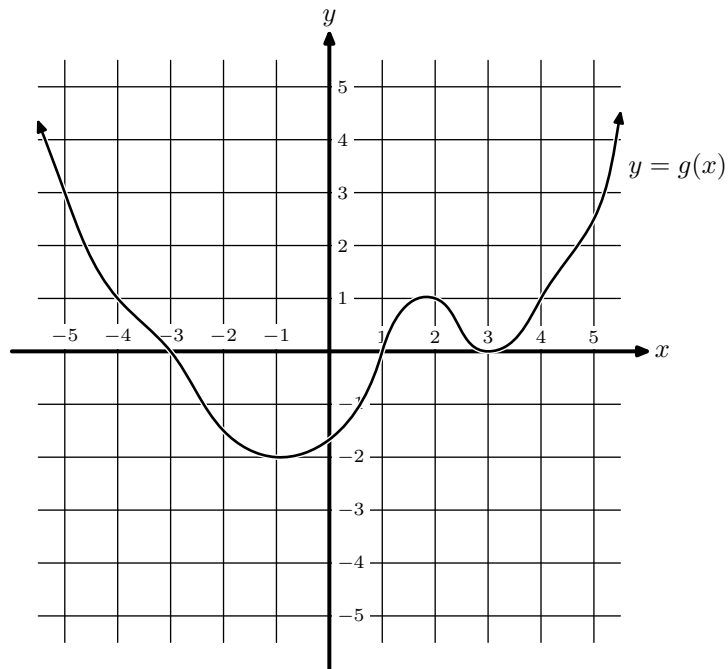
(a) (8 points) Find the value of  $k$  in the exponential decay model above.

(b) (8 points) What is the half-life of rhenium-182?

**Problem 7.** (12 points)

(a) (6 points) What is the domain of  $f(x) = \ln x$ ?

(b) (6 points) The graph of a function  $g(x)$  is shown below. Based on this graph, what is the domain of the function  $h(x) = \ln(g(x))$ ? Why?



**Problem 8.** (10 points) Solve

$$\log(3x + 157) + \log(100) = 5.$$

**Bonus problem** (+4 points). There are a few trigonometric functions which were once common but are now very rarely used. One of these is called the *versine*, written  $\text{versin } \theta$ , which is defined as

$$\text{versin } \theta = 1 - \cos \theta.$$

Find the exact value of  $(1 + \sin 85^\circ)(\text{versin } 5^\circ)(\csc^2 5^\circ)$  without using a calculator.