

NAME:

MATH 103 Exam 3, version (a)

24 October 2008

100 points

Instructions:

1. This exam has 6 pages (including this one), which contain 8 problems and one bonus problem. Please check that you have all of the pages.
 2. Answer all of the following questions clearly and completely. Justify all of your answers.
 3. You may not use a book or any notes for this exam.
 4. Give your answer to each problem completely and clearly in the space provided. You may use the back of the exam pages for scratch work; however, if you want this work to be considered, make note of it in the space provided for the problem.
 5. Erase or cross out work you do not wish to be graded.
 6. Credit, partial or full, will be given only if sufficient steps leading to the answers are shown.
 7. You have 50 minutes to complete this exam.
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Problem 1. (10 points) Be sure to show work for both parts of this problem, even if you can do them in your head, so that I can see that you understand how to do these conversions and that you aren't just using your calculator.

(a) (5 points) Convert $\frac{3\pi}{4}$ radians to degrees.

(b) (5 points) Convert 75° to radians. Express your answer as a multiple of π (in other words, do not approximate your answer with a decimal).

Problem 2. (12 points) Find the exact value of $2\log_7 21 - \log_7 9$ without using a calculator.

Problem 3. (12 points) Suppose θ is an acute angle and $\cos \theta = \frac{40}{41}$. Find the values of the other five trigonometric functions of θ .

Problem 4. (14 points) The function $f(x) = \frac{4}{2-x}$ is one-to-one. Find its inverse function f^{-1} and check your answer.

Problem 5. (10 points) Solve

$$5 \log(x + 571) = 15.$$

Problem 6. (14 points) You have \$850 to invest. What rate of return r , compounded continuously, do you need in order to have \$1250 after 6 years?

Problem 7. (16 points) In an electric circuit, the voltage across the terminals of a capacitor which is discharging through a resistor decays exponentially. In other words, the voltage $V(t)$ at time t is given by

$$V(t) = V_0 e^{kt},$$

where V_0 is the voltage at time $t = 0$. Suppose this capacitor is charged so that the voltage across its terminals at time $t = 0$ is 25 volts. After 10 seconds the voltage is measured and is found to be 8 volts.

(a) (8 points) Find the value of k in the exponential decay model above.

(b) (8 points) What will be the voltage across the terminals of the capacitor at time $t = 15$ seconds?

Problem 8. (12 points) A ladder is leaning against a wall. The base of the ladder is 4 feet from the base of the wall, and the ladder makes an angle of 70.5° with the floor. How long is the ladder?

Bonus problem. (+4 points) There are a few trigonometric functions which were once common but are now very rarely used. One of these is called the *versine*, written $\text{versin } \theta$, which is defined as

$$\text{versin } \theta = 1 - \cos \theta.$$

Find the exact value of $(1 + \sin 68^\circ)(\text{versin } 22^\circ)(\csc^2 22^\circ)$ without using a calculator.